Basic College Mathematics

EDITION 10

AN APPLIED APPROACH

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SECTION

12.1 Angles, Lines, and Geometric Figures

OBJECTIVE A *To define and describe lines and angles*

Point of Interest

Geometry is one of the oldest branches of mathematics. Around 350 B.C., the Greek mathematician Euclid wrote the *Elements*, which contained all of the known concepts of geometry. Euclid's contribution was to unify various concepts into a single deductive system that was based on a set of axioms.

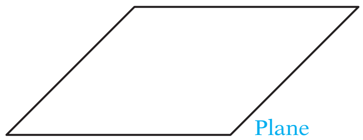
Tips for Success

A great many new vocabulary words are introduced in this chapter. There are eight new terms on this page alone: plane, plane figures, space, solids, line, line segment, parallel lines, and intersecting lines. All of these terms are in **bold type.** The bold type indicates that these are concepts you must know to learn the material. Be sure to study each new term as it is presented.

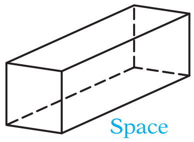
The word *geometry* comes from the Greek words for "earth" (*geo*) and "measure" (*metron*). The original purpose of geometry was to measure land. Today geometry is used in many sciences, such as physics, chemistry, and geology, and in applied fields such as mechanical drawing and astronomy. Geometric forms are used in art and design.

Two basic geometric concepts are plane and space.

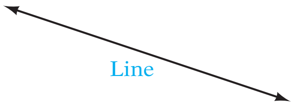
A **plane** is a flat surface, such as a table top or a blackboard. Figures that lie totally in a plane are called **plane figures.**



Space extends in all directions. Objects in space, such as trees, ice cubes, and doors, are called **solids.**

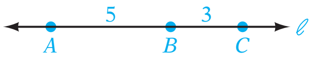


A **line** extends indefinitely in two directions in a plane. A line has no width.



A **line segment** is part of a line and has two endpoints. The line segment, written as $\overline{AB}$, is shown in the figure.

The length of a line segment is the distance between the endpoints of the line segment. The length of a line segment may be expressed as the sum of two or more shorter line segments. For the figure at the right, $AB = 5$, $BC = 3$, and $AC = AB + BC = 5 + 3 = 8$.



Take Note

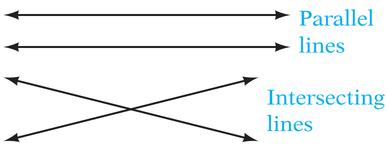
When no units, such as feet or meters, are given for the lengths along a line segment, all of the distances are assumed to be in the same unit of length.

HOW TO 1 Given that $AB = 22$ and $AC = 31$, find BC.

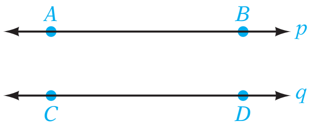
$$AC = AB + BC$$
$$31 = 22 + BC$$
$$31 - 22 = 22 - 22 + BC$$
$$9 = BC$$

• **Substitute 22 for AB and 31 for AC, and solve for BC.**

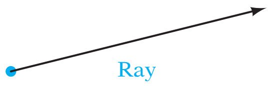
Lines in a plane can be parallel or intersecting. **Parallel lines** never meet; the distance between them is always the same. **Intersecting lines** cross at a point in the plane.



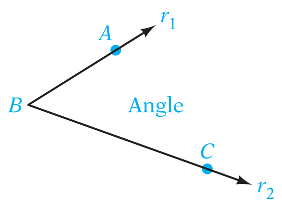
The symbol $\parallel$ means "is parallel to." In the accompanying figure, $\overline{AB} \parallel \overline{CD}$ and $p \parallel q$. Note that line p contains line segment AB and line q contains line segment CD. Parallel lines contain parallel line segments.



A **ray** starts at a point and extends indefinitely in one direction.



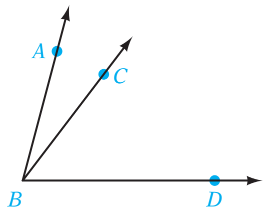
An **angle** is formed when two rays start from the same point. Rays r_1 and r_2 start from point B. The common endpoint is called the **vertex** of the angle.



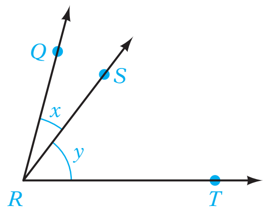
If A and C are points on rays r_1 and r_2 above, respectively, then the angle is called $\angle ABC$, $\angle CBA$, or $\angle B$, where $\angle$ is the symbol for angle. Note that an angle is named by giving three points, with the vertex as the second point listed, or by giving the point at the vertex.

Take Note

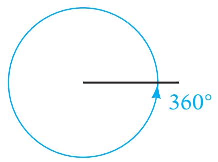
For the figure below, we can refer to $\angle ABD$, $\angle ABC$, or $\angle CBD$. Just writing $\angle B$ does not identify a specific angle.



An angle can also be named by writing a variable between the rays, close to the vertex. In the figure, $\angle x = \angle QRS = \angle SRQ$ and $\angle y = \angle SRT = \angle TRS$. Note that in this figure, more than two rays meet at the vertex. In this case, the vertex cannot be used to name the angle.



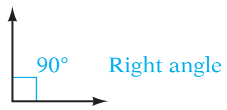
A unit in which angles are measured is the **degree.** The symbol for degree is °. One complete revolution is 360° (360 degrees).



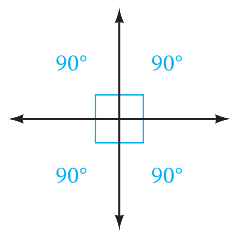
Point of Interest

The Babylonians knew that Earth is in approximately the same position in the sky every 365 days. Historians suggest that the reason one complete revolution of a circle is 360° is because 360 is the closest number to 365 that is divisible by many numbers.

One-quarter of a revolution is 90°. A 90° angle is called a **right angle.** The symbol ⊾ represents a right angle.



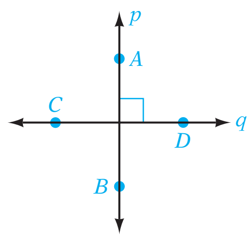
Perpendicular lines are intersecting lines that form right angles.



Take Note

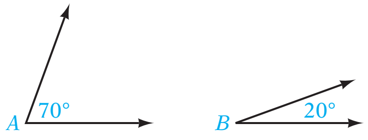
The corner of a page of this book serves as a good model for a 90° angle.

The symbol $\perp$ means "is perpendicular to." In the accompanying figure, $\overline{AB} \perp \overline{CD}$ and $p \perp q$. Note that line p contains line segment AB and line q contains line segment CD. Perpendicular lines contain perpendicular line segments.

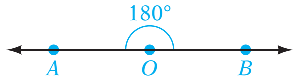


Complementary angles are two angles whose sum is $90°$.

$\angle A + \angle B = 70° + 20° = 90°$
$\angle A$ and $\angle B$ are complementary angles.

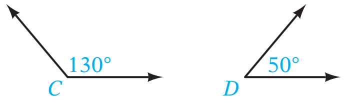


One-half of a revolution is 180°. A 180° angle is called a **straight angle.** $\angle AOB$ in the figure is a straight angle.



Supplementary angles are two angles whose sum is $180°$.

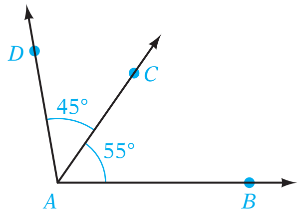
$\angle C + \angle D = 130° + 50° = 180°$
$\angle C$ and $\angle D$ are supplementary angles.



An **acute angle** is an angle whose measure is between 0° and 90°. $\angle D$ in the figure above is an acute angle. An **obtuse angle** is an angle whose measure is between 90° and 180°. $\angle C$ in the figure above is an obtuse angle.

In the accompanying figure, $\angle DAC = 45°$ and $\angle CAB = 55°$.

$$\begin{aligned} \angle DAB &= \angle DAC + \angle CAB \\ &= 45° + 55° \\ &= 100° \end{aligned}$$



EXAMPLE 1

Given that $MN = 15$, $NO = 18$, and $MP = 48$, find OP.

Solution

$$\begin{aligned} MP &= MN + NO + OP \\ 48 &= 15 + 18 + OP \\ 48 &= 33 + OP \\ 48 - 33 &= 33 - 33 + OP \\ 15 &= OP \end{aligned}$$

YOU TRY IT 1

Given that $QR = 24$, $ST = 17$, and $QT = 62$, find RS.

Your solution

EXAMPLE 2

Find the complement of a 32° angle.

Solution

Let x represent the complement of 32°.

$$\begin{aligned} x + 32° &= 90° \\ x + 32° - 32° &= 90° - 32° \\ x &= 58° \end{aligned}$$

• **The sum of complementary angles is 90°.**

58° is the complement of 32°.

YOU TRY IT 2

Find the supplement of a 32° angle.

Your solution

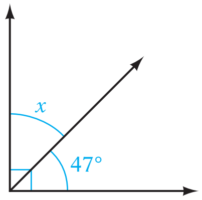
Solutions on pp. S28–S29

EXAMPLE 3

Find the measure of $\angle x$.

Solution

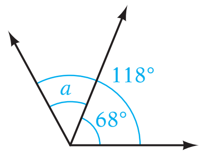
$$\angle x + 47° = 90°$$
$$\angle x + 47° - 47° = 90° - 47°$$
$$\angle x = 43°$$



YOU TRY IT 3

Find the measure of $\angle a$.

Your solution



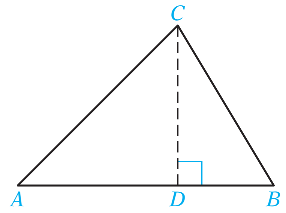
Solution on p. S29

OBJECTIVE B *To define and describe geometric figures*

Take Note

Any side of a triangle can be considered the base, but generally the **base of a triangle** refers to the side that the triangle rests on. The **height of a triangle** is a line segment drawn perpendicular to the base from the opposite vertex.

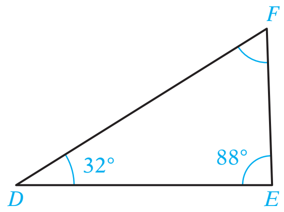
A **triangle** is a closed, three-sided plane figure. Figure *ABC* is a triangle. $\overline{AB}$ is called the **base.** The line $\overline{CD}$, perpendicular to the base, is called the **height.**



The Angles of a Triangle

The sum of the three angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

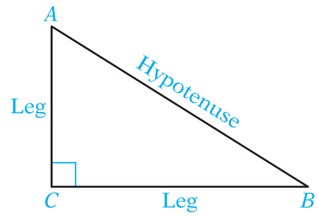


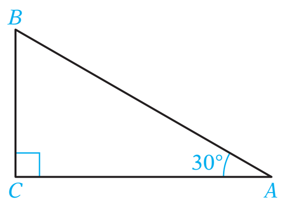
HOW TO 2 In triangle *DEF*, $\angle D = 32°$ and $\angle E = 88°$. Find the measure of $\angle F$.

$$\angle D + \angle E + \angle F = 180°$$ • The sum of the three angles of a triangle is 180°.
$$32° + 88° + \angle F = 180°$$ • $\angle D = 32°$ and $\angle E = 88°$
$$120° + \angle F = 180°$$ • Solve for $\angle F$.
$$120° - 120° + \angle F = 180° - 120°$$
$$\angle F = 60°$$

A **right triangle** contains one right angle. The side opposite the right angle is called the **hypotenuse.** The **legs of a right triangle** are its other two sides. In a right triangle, the two acute angles are complementary.

$\angle A + \angle B = 90°$



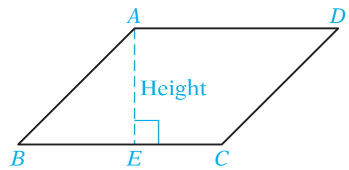


HOW TO 3 In the right triangle at the left, $\angle A = 30°$. Find the measure of $\angle B$.

$$\angle A + \angle B = 90°$$ • The two acute angles are complementary.
$$30° + \angle B = 90°$$ • $\angle A = 30°$
$$30° - 30° + \angle B = 90° - 30°$$ • Solve for $\angle B$.
$$\angle B = 60°$$

A **quadrilateral** is a closed, four-sided plane figure. Three quadrilaterals with special characteristics are described here.

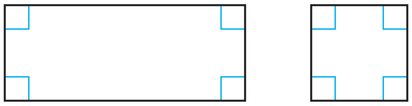
A **parallelogram** has opposite sides parallel and equal. The perpendicular distance *AE* between the parallel sides is called the **height.**



Parallelogram

A **rectangle** is a parallelogram that has four right angles.

A **square** is a rectangle that has four equal sides.

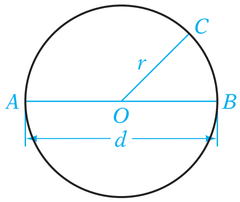


Rectangle Square

A **circle** is a plane figure in which all points are the same distance from point *O*, which is called the **center** of the circle.

The **diameter of a circle** (d) is the length of a line segment through the center of the circle with endpoints on the circle. *AB* is a diameter of the circle shown.

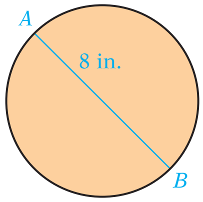
The **radius of a circle** (r) is the length of a line segment from the center to a point on the circle. *OC* is a radius of the circle shown.



Circle

$$d = 2r \quad \text{or} \quad r = \frac{1}{2}d$$

HOW TO 4 The line segment *AB* is a diameter of the circle shown. Find the radius of the circle.



The radius is one-half the diameter. Therefore,

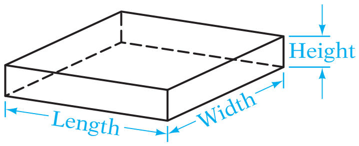
$$r = \frac{1}{2}d$$

$$= \frac{1}{2}(8 \text{ in.}) \qquad \bullet \ d = 8 \text{ in.}$$

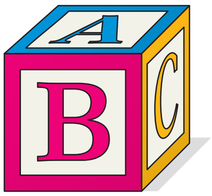
$$= 4 \text{ in.}$$

A **geometric solid** is a figure in space, or space figure. Four common space figures are the rectangular solid, cube, sphere, and cylinder.

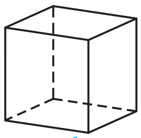
A **rectangular solid** is a solid in which all six faces are rectangles.



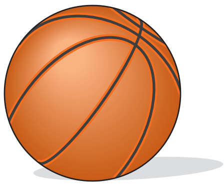
Rectangular solid



A **cube** is a rectangular solid in which all six faces are squares.

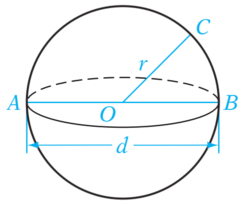


Cube



A **sphere** is a solid in which all points on the surface are the same distance from point *O*, which is called the **center** of the sphere.

The **diameter of a sphere** is the length of a line segment going through the center with endpoints on the sphere. *AB* is a diameter of the sphere shown.

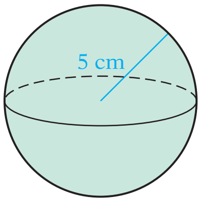


Sphere

The **radius of a sphere** is the length of a line segment from the center to a point on the sphere. *OC* is a radius of the sphere shown above.

$$d = 2r \quad \text{or} \quad r = \frac{1}{2}d$$

HOW TO 5 The radius of the sphere shown at the right is 5 cm. Find the diameter of the sphere.



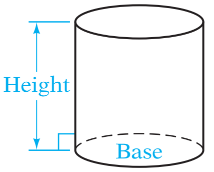
$d = 2r$ • **The diameter equals twice the radius.**

$= 2(5 \text{ cm})$ • $r = 5$ **cm**

$= 10 \text{ cm}$

The diameter is 10 cm.

The most common **cylinder** is one in which the bases are circles and are perpendicular to the side.



Cylinder

EXAMPLE 4

One angle in a right triangle measures 50°. Find the measures of the other two angles.

Solution

In a right triangle, one angle measures 90° and the two acute angles are complementary.

$$\angle A + \angle B = 90°$$
$$\angle A + 50° = 90°$$
$$\angle A + 50° - 50° = 90° - 50°$$
$$\angle A = 40°$$

The other angles measure 90° and 40°.

YOU TRY IT 4

A right triangle has one angle measuring 7°. Find the measures of the other two angles.

Your solution

EXAMPLE 5

Two angles of a triangle measure 42° and 103°. Find the measure of the third angle.

Solution

The sum of the three angles of a triangle is 180°.

$$\angle A + \angle B + \angle C = 180°$$
$$\angle A + 42° + 103° = 180°$$
$$\angle A + 145° = 180°$$
$$\angle A + 145° - 145° = 180° - 145°$$
$$\angle A = 35°$$

The measure of the third angle is 35°.

YOU TRY IT 5

Two angles of a triangle measure 62° and 45°. Find the measure of the third angle.

Your solution

EXAMPLE 6

A circle has a radius of 8 cm. Find the diameter.

Solution

$d = 2r$

$= 2 \cdot 8 \text{ cm} = 16 \text{ cm}$

The diameter is 16 cm.

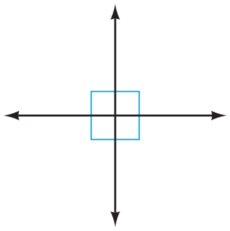
YOU TRY IT 6

A circle has a diameter of 8 in. Find the radius.

Your solution

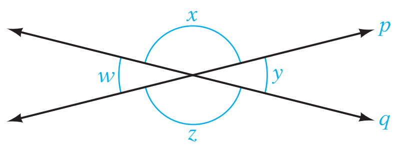
Solutions on p. S29

OBJECTIVE C *To solve problems involving angles formed by intersecting lines*



Four angles are formed by the intersection of two lines. If the two lines are perpendicular, then each of the four angles formed is a right angle. If the two lines are not perpendicular, then two of the angles formed are acute angles and two of the angles formed are obtuse angles. The two acute angles are always opposite each other, and the two obtuse angles are always opposite each other.

In the figure, $\angle w$ and $\angle y$ are acute angles. $\angle x$ and $\angle z$ are obtuse angles. Two angles that are on opposite sides of the intersection of two lines are called **vertical angles.** Vertical angles have the same measure. $\angle w$ and $\angle y$ are vertical angles. $\angle x$ and $\angle z$ are vertical angles.

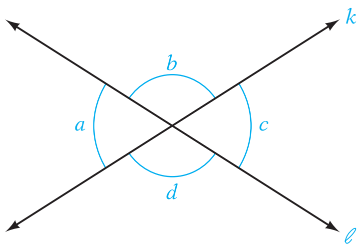


$\angle w = \angle y$
$\angle x = \angle z$

Two angles that share a common side are called **adjacent angles.** In the figure above, $\angle x$ and $\angle y$ are adjacent angles, as are $\angle y$ and $\angle z$, $\angle z$ and $\angle w$, and $\angle w$ and $\angle x$. Adjacent angles of intersecting lines are supplementary angles.

$\angle x + \angle y = 180°$
$\angle y + \angle z = 180°$
$\angle z + \angle w = 180°$
$\angle w + \angle x = 180°$

HOW TO 6 In the figure at the left, $\angle c = 65°$. Find the measures of angles *a, b,* and *d.*



$\angle a = 65°$ • **$\angle a = \angle c$ because $\angle c$ and $\angle a$ are vertical angles.**

$\angle b + \angle c = 180°$ • **$\angle c$ is supplementary to $\angle b$ because $\angle c$ and $\angle b$ are adjacent angles.**

$\angle b + 65° = 180°$ • **$\angle c = 65°$**

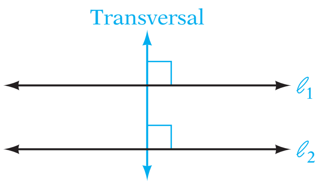
$\angle b + 65° - 65° = 180° - 65°$

$\angle b = 115°$

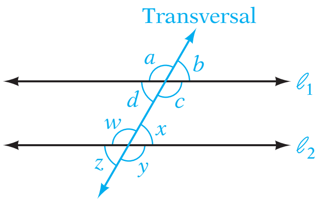
$\angle d = 115°$ • **$\angle d = \angle b$ because $\angle b$ and $\angle d$ are vertical angles.**

A line intersecting two other lines at two different points is called a **transversal.**

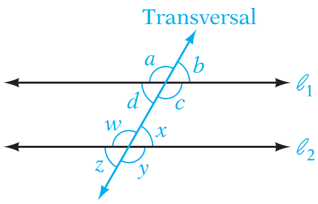
If the lines cut by a transversal are parallel lines and the transversal is perpendicular to the parallel lines, then all eight angles formed are right angles.



If the lines cut by a transversal are parallel lines and the transversal is not perpendicular to the parallel lines, then all four acute angles have the same measure and all four obtuse angles have the same measure. For the figure at the right, there are two groups of angles with the same measure:



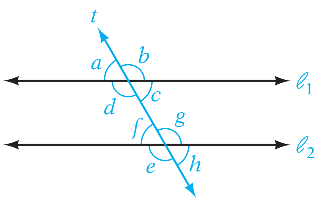
$\angle a = \angle c = \angle w = \angle y$ and $\angle b = \angle d = \angle x = \angle z$



Alternate interior angles are two nonadjacent angles that are on opposite sides of the transversal and between the parallel lines. For the figure at the left, $\angle c$ and $\angle w$ are alternate interior angles. $\angle d$ and $\angle x$ are alternate interior angles. Alternate interior angles have the same measure.

Alternate exterior angles are two nonadjacent angles that are on opposite sides of the transversal and outside the parallel lines. For the figure at the left, $\angle a$ and $\angle y$ are alternate exterior angles. $\angle b$ and $\angle z$ are alternate exterior angles. Alternate exterior angles have the same measure.

Corresponding angles are two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. For the figure at the top left, the following pairs of angles are corresponding angles: $\angle a$ and $\angle w$, $\angle d$ and $\angle z$, $\angle b$ and $\angle x$, $\angle c$ and $\angle y$. Corresponding angles have the same measure.



HOW TO 7 In the figure at the left, $\ell_1 \parallel \ell_2$ and $\angle c = 58°$. Find the measures of $\angle f$, $\angle h$, and $\angle g$.

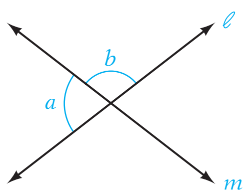
$\angle f = 58°$ • **$\angle f = \angle c$ because $\angle f$ and $\angle c$ are alternate interior angles.**

$\angle h = 58°$ • **$\angle h = \angle c$ because $\angle c$ and $\angle h$ are corresponding angles.**

$\angle g + \angle h = 180°$ • **$\angle g$ is supplementary to $\angle h$.**
$\angle g + 58° = 180°$ • **$\angle h = 58°$**
$\angle g = 122°$ • **Subtract 58° from each side.**

EXAMPLE 7

In the figure, $\angle a = 75°$.
Find $\angle b$.

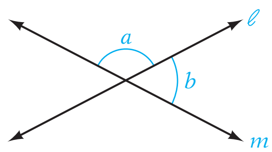


Solution

$\angle a + \angle b = 180°$ • **$\angle a$ and $\angle b$ are supplementary.**
$75° + \angle b = 180°$ • **$\angle a = 75°$**
$\angle b = 105°$ • **Subtract 75° from each side.**

YOU TRY IT 7

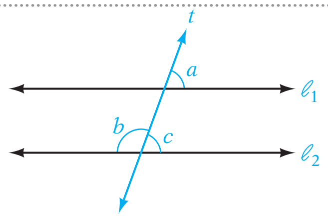
In the figure, $\angle a = 125°$.
Find $\angle b$.



Your solution

EXAMPLE 8

In the figure, $\ell_1 \parallel \ell_2$ and $\angle a = 70°$.
Find $\angle b$.



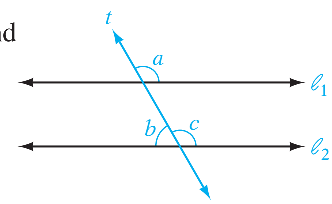
Solution

$\angle c = \angle a = 70°$ • **Corresponding angles are equal.**

$\angle b + \angle c = 180°$ • **$\angle b$ and $\angle c$ are supplementary.**
$\angle b + 70° = 180°$ • **$\angle c = 70°$**
$\angle b = 110°$ • **Subtract 70° from each side.**

YOU TRY IT 8

In the figure, $\ell_1 \parallel \ell_2$ and $\angle a = 120°$.
Find $\angle b$.



Your solution

Solutions on p. S29

12.1 EXERCISES

Concept Check

1. The measure of an acute angle is between ___ and ___.
2. The measure of an obtuse angle is between ___ and ___.
3. How many degrees are in a straight angle?
4. How many degrees are in a right angle?
5. Two lines that intersect at right angles are ___________ lines.
6. What is the sum of the three angles of a triangle?
7. Name the side opposite the right angle in a right triangle.
8. Which sides are the legs of a right triangle?

OBJECTIVE A *To define and describe lines and angles*

9. In the figure, $EF = 20$ and $FG = 10$. Find EG.

10. In the figure, $EF = 18$ and $FG = 6$. Find EG.

11. In the figure, $QR = 7$ and $QS = 28$. Find RS.

12. In the figure, $QR = 15$ and $QS = 45$. Find RS.

13. In the figure, $AB = 12$, $CD = 9$, and $AD = 35$. Find BC.

14. In the figure, $AB = 21$, $BC = 14$, and $AD = 54$. Find CD.

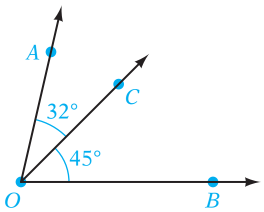
15. Find the complement of a 31° angle.
16. Find the complement of a 62° angle.
17. Find the supplement of a 72° angle.
18. Find the supplement of a 162° angle.
19. Find the complement of a 13° angle.
20. Find the complement of an 88° angle.
21. Find the supplement of a 127° angle.
22. Find the supplement of a 7° angle.

For Exercises 23 to 26, determine whether the described angle is an acute angle, is a right angle, is an obtuse angle, or does not exist.

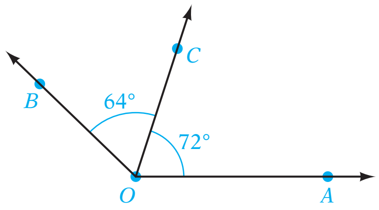
23. The complement of an acute angle
24. The supplement of a right angle
25. The supplement of an acute angle
26. The complement of an obtuse angle

For Exercises 27 and 28, find the measure of angle *AOB*.

27.

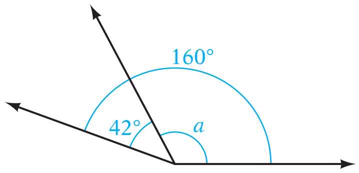


28.

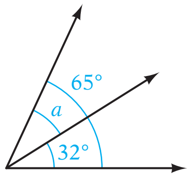


For Exercises 29 to 32, find the measure of angle *a*.

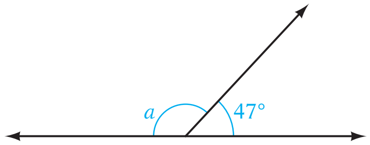
29.



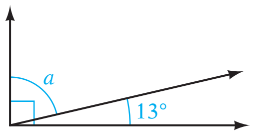
30.



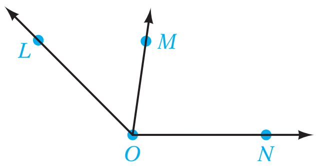
31.



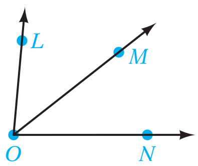
32.



33. In the figure, $\angle LOM = 53°$ and $\angle LON = 139°$. Find the measure of $\angle MON$.



34. In the figure, $\angle MON = 38°$ and $\angle LON = 85°$. Find the measure of $\angle LOM$.



OBJECTIVE B *To define and describe geometric figures*

35. Name the rectangle with four equal sides.

36. Name the solid in which all points are the same distance from the center.

37. Name the plane figure in which all points are the same distance from the center.

38. Name the solid in which the bases are circular and perpendicular to the side.

39. A triangle has a 13° angle and a 65° angle. Find the measure of the other angle.

40. A triangle has a 105° angle and a 32° angle. Find the measure of the other angle.

41. A right triangle has a 45° angle. Find the measures of the other two angles.

42. A right triangle has a 62° angle. Find the measures of the other two angles.

43. A triangle has a 62° angle and a 104° angle. Find the measure of the other angle.

44. A triangle has a 30° angle and a 45° angle. Find the measure of the other angle.

45. A right triangle has a 25° angle. Find the measures of the other two angles.

46. Two angles of a triangle measure 42° and 105°. Find the measure of the other angle.

47. Find the radius of a circle with a diameter of 16 in.

48. Find the radius of a circle with a diameter of 9 ft.

49. Find the diameter of a circle with a radius of $2\frac{1}{3}$ ft.

50. Find the diameter of a circle with a radius of 24 cm.

51. The radius of a sphere is 3.5 cm. Find the diameter.

52. The radius of a sphere is $1\frac{1}{2}$ ft. Find the diameter.

53. The diameter of a sphere is 4 ft 8 in. Find the radius.

54. The diameter of a sphere is 1.2 m. Find the radius.

For Exercises 55 to 57, determine whether the statement is true or false.

55. A triangle can have two obtuse angles.

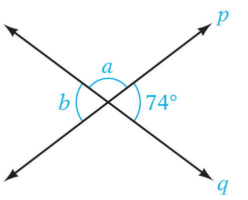
56. The legs of a right triangle are perpendicular.

57. If the sum of two angles of a triangle is less than 90°, then the third angle is an obtuse angle.

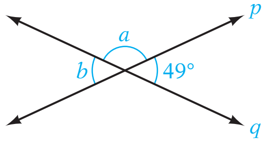
OBJECTIVE C *To solve problems involving angles formed by intersecting lines*

For Exercises 58 to 61, find the measures of angles *a* and *b*.

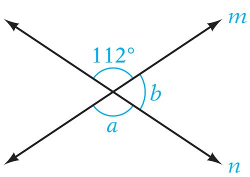
58.



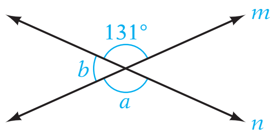
59.



60.

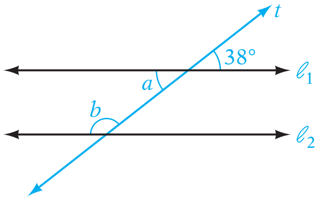


61.

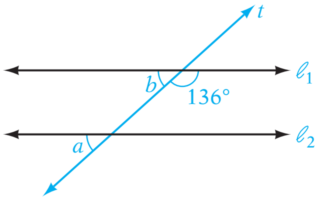


For Exercises 62 to 69, $\ell_1 \| \ell_2$. Find the measures of angles a and b.

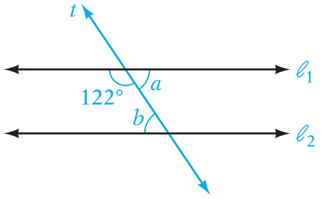
62.



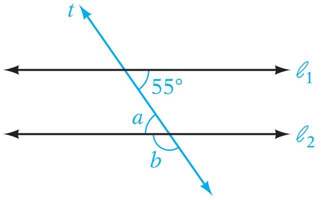
63.



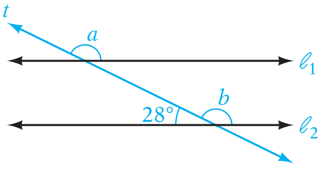
64.



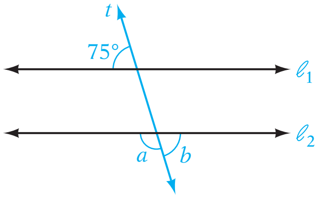
65.



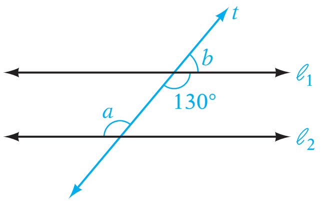
66.



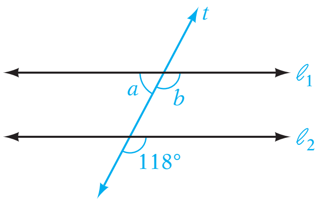
67.



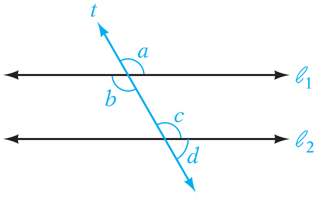
68.



69.



For Exercises 70 to 72, use the diagram at the right. Determine whether the given statement is true or false.



70. $\angle a$ and $\angle b$ have the same measure even if ℓ_1 and ℓ_2 are not parallel.

71. $\angle c$ and $\angle d$ are supplementary even if ℓ_1 and ℓ_2 are not parallel.

72. If $\angle a$ is greater than $\angle c$, then ℓ_1 and ℓ_2 are not parallel.

Critical Thinking

73. If AB and CD intersect at point O, and $\angle AOC = \angle BOC$, explain why AB is perpendicular to CD.

Projects or Group Activities

74. In a plane, the shortest distance between two points is a straight line. What is the shortest distance between two points on a sphere? Earth is approximately spherical in shape. What are some practical applications of knowing how to find the shortest distance between two points on a sphere?

SECTION

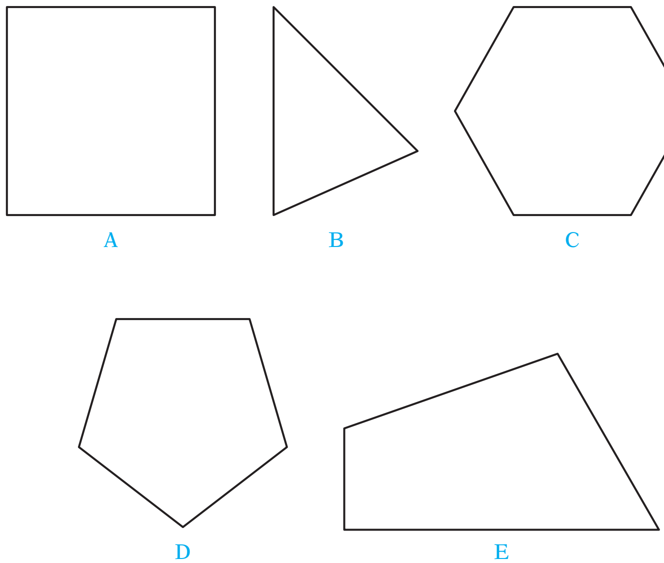
12.2 Plane Geometric Figures

OBJECTIVE A *To find the perimeter of plane geometric figures*

A **polygon** is a closed figure determined by three or more line segments that lie in a plane. The **sides of a polygon** are the line segments that form the polygon. The figures below are examples of polygons.

Point of Interest

Although a polygon is defined in terms of its *sides* (see the definition above), the word *polygon* actually comes from the Latin word *polygonum,* which means "having many *angles.*" This is certainly the case for a polygon.



A **regular polygon** is one in which each side has the same length and each angle has the same measure. The polygons in Figures A, C, and D above are regular polygons.

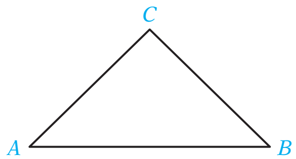
Frontpage/Shutterstock.com

The Pentagon in Arlington, Virginia

The name of a polygon is based on the number of its sides. The table below lists the names of polygons that have from 3 to 10 sides.

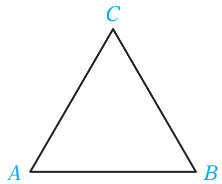
Number of Sides	*Name of the Polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Triangles and quadrilaterals are two of the most common types of polygons. Triangles are distinguished by the number of equal sides and also by the measures of their angles.



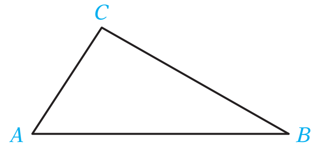
An **isosceles triangle** has two sides of equal length. The angles opposite each of the equal sides are of equal measure.

$AC = BC$
$\angle A = \angle B$

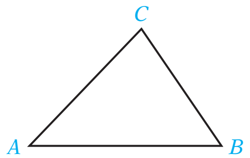


The three sides of an **equilateral triangle** are of equal length. The three angles are of equal measure.

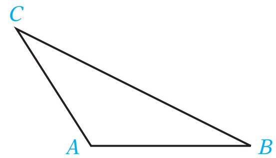
$AB = BC = AC$
$\angle A = \angle B = \angle C$



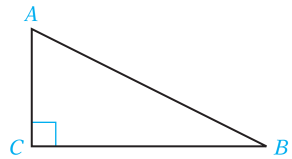
A **scalene triangle** has no two sides of equal length. No two angles are of equal measure.



An **acute triangle** has three acute angles.

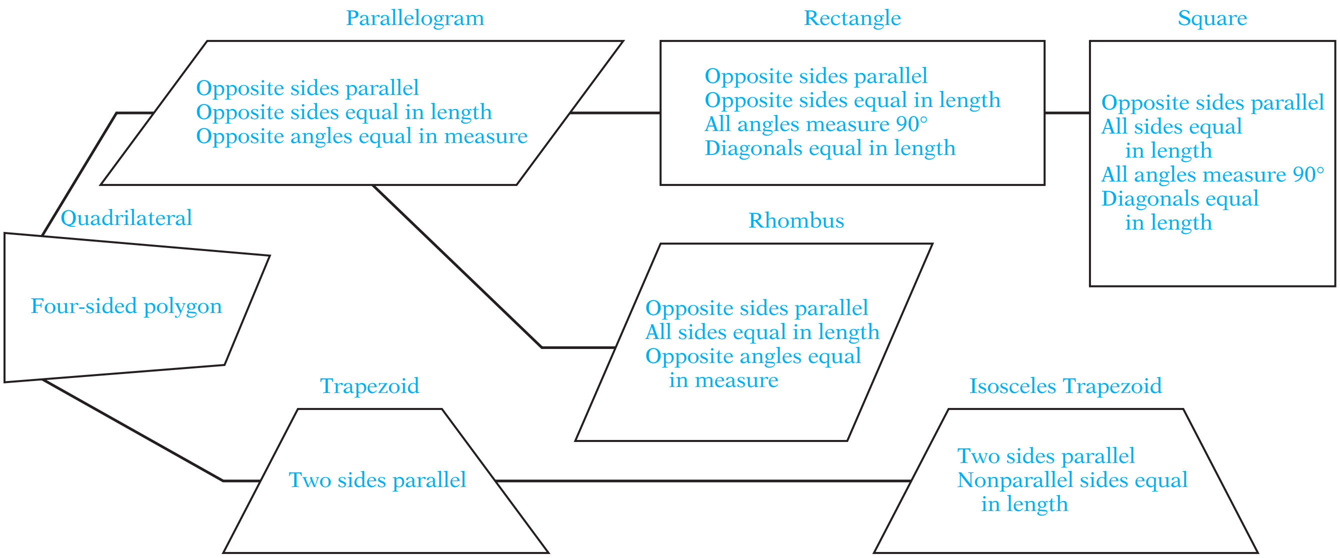


An **obtuse triangle** has one obtuse angle.



A **right triangle** has a right angle.

Quadrilaterals also are distinguished by their sides and angles, as shown below. Note that a rectangle, a square, and a rhombus are different forms of a parallelogram.



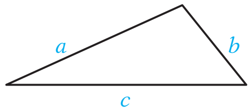
The **perimeter** of a plane geometric figure is a measure of the distance around the figure. The perimeter of a polygon is the sum of the lengths of its sides. Perimeter is used, for example, in buying fencing for a lawn or in determining how much baseboard is needed for a room.

Here are the perimeter formulas for some of the more common geometric figures.

Perimeter of a Triangle

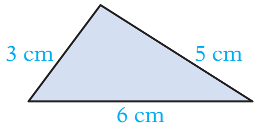
Let a, b, and c be the lengths of the sides of a triangle.

The perimeter of the triangle is $P = a + b + c$.



EXAMPLE

Find the perimeter of the triangle shown at the right.



$P = a + b + c$

$= 3 \text{ cm} + 5 \text{ cm} + 6 \text{ cm}$

$= 14 \text{ cm}$

The perimeter of the triangle is 14 cm.

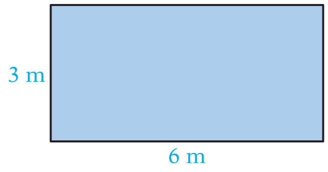
Perimeter of a Rectangle

Let L be the length (usually the longer side) of a rectangle and W be the width (usually the shorter side) of a rectangle.

The perimeter of the rectangle is $P = 2L + 2W$.

EXAMPLE

Find the perimeter of the rectangle shown at the right.



$P = 2L + 2W$

$= 2(6 \text{ m}) + 2(3 \text{ m})$ • $L = 6$ m; $W = 3$ m

$= 12 \text{ m} + 6 \text{ m}$

$= 18 \text{ m}$

The perimeter of the rectangle is 18 m.

APPLY THE CONCEPT

A building contractor must place a security fence around a rectangular construction lot that is 95 ft long and 72 ft wide. How many feet of fencing must the contractor buy?

To find the amount of fencing needed, find the perimeter of the lot.

$P = 2L + 2W$

$= 2(95 \text{ ft}) + 2(72 \text{ ft})$ • $L = 95$ ft; $W = 72$ ft

$= 190 \text{ ft} + 144 \text{ ft}$

$= 334 \text{ ft}$

The contractor must buy 334 ft of fencing.

Take Note

The perimeter of a square is the sum of the four sides:

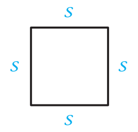
$s + s + s + s = 4s$

Recall that a square is a rectangle in which all four sides are equal.

Perimeter of a Square

Let s be the length of a side of a square.

The perimeter of the square is $P = 4s$.



EXAMPLE

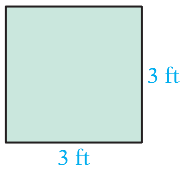
Find the perimeter of the square shown at the right.

$P = 4s$

$= 4(3 \text{ ft})$ • $s = 3$ ft

$= 12$ ft

The perimeter of the square is 12 ft.



Point of Interest

Archimedes (c. 287–212 B.C.) was the mathematician who gave us the approximate value of π as $\frac{22}{7} = 3\frac{1}{7}$. He actually showed that π was between $3\frac{10}{71}$ and $3\frac{1}{7}$. The approximation $3\frac{10}{71}$ is closer to the exact value of π, but it is more difficult to use.

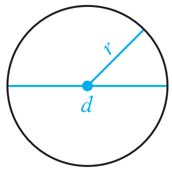
The perimeter of a circle is called its **circumference.** The circumference of a circle is equal to the product of pi (π) and the diameter. The value of π can be approximated as

$$\pi \approx 3.14 \text{ or } \pi \approx \frac{22}{7}$$

The π key on a calculator gives a more exact approximation of π.

Circumference of a Circle

Let d be the diameter of a circle.
The circumference of the circle is $C = \pi d$.
Because the diameter is twice the radius, the circumference is also given by $C = 2\pi r$.



EXAMPLE

Find the circumference of the circle shown at the right.

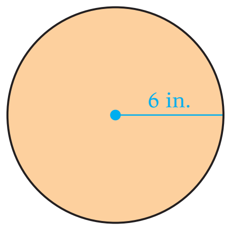
$C = 2\pi r$

$= 2\pi(6 \text{ in.})$ • $r = 6$ in.

$\approx 2(3.14)(6 \text{ in.})$

$= 37.68$ in.

The circumference of the circle is approximately 37.68 in.



APPLY THE CONCEPT

The diameter of a car tire is 16 in. If the tire makes two complete revolutions, what distance has the tire traveled? Use 3.14 for π.

To find the distance, first find the circumference of the tire. Because the tire makes two complete revolutions, multiply the circumference by 2.

$C = \pi d$

$\approx 3.14(16 \text{ in.})$ • $d = 16$ in.

$= 50.24$ in.

The circumference of the tire is approximately 50.24 in.

$2 \times 50.24 = 100.48$

The tire has traveled a distance of 100.48 in.

EXAMPLE 1

Find the perimeter of a rectangle with a width of $\frac{2}{3}$ ft and a length of 2 ft.

Solution

$$P = 2L + 2W$$
$$= 2(2\text{ ft}) + 2\left(\frac{2}{3}\text{ ft}\right) \quad \bullet\ L = 2\text{ ft}, W = \frac{2}{3}\text{ ft}$$
$$= 4\text{ ft} + \frac{4}{3}\text{ ft}$$
$$= 5\frac{1}{3}\text{ ft}$$

The perimeter of the rectangle is $5\frac{1}{3}$ ft.

YOU TRY IT 1

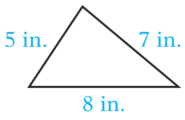
Find the perimeter of a rectangle with a length of 2 m and a width of 0.85 m.

Your solution

EXAMPLE 2

Find the perimeter of a triangle with sides of lengths 5 in., 7 in., and 8 in.

Solution



$$P = a + b + c$$
$$= 5\text{ in.} + 7\text{ in.} + 8\text{ in.}$$
$$= 20\text{ in.}$$

The perimeter of the triangle is 20 in.

YOU TRY IT 2

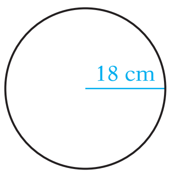
Find the perimeter of a triangle with sides of lengths 12 cm, 15 cm, and 18 cm.

Your solution

EXAMPLE 3

Find the circumference of a circle with a radius of 18 cm. Use 3.14 for π.

Solution



$$C = 2\pi r$$
$$\approx 2 \cdot 3.14 \cdot 18\text{ cm}$$
$$= 113.04\text{ cm}$$

The circumference is approximately 113.04 cm.

YOU TRY IT 3

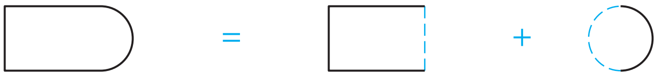
Find the circumference of a circle with a diameter of 6 in. Use 3.14 for π.

Your solution

Solutions on p. S29

OBJECTIVE B *To find the perimeter of composite geometric figures*

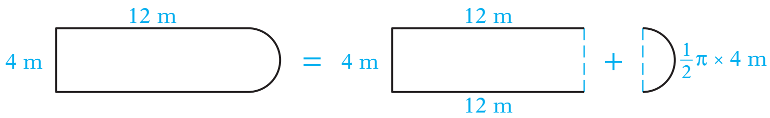
A **composite geometric figure** is a figure made from two or more geometric figures. The following composite is made from part of a rectangle and part of a circle:



Perimeter of the composite figure $= 3$ sides of a rectangle $+ \frac{1}{2}$ the circumference of a circle

Perimeter of the composite figure $= 2L + W + \frac{1}{2}\pi d$

The perimeter of the composite figure below is found by adding twice the length plus the width plus one-half the circumference of the circle.



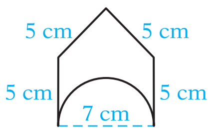
$$P = 2L + W + \frac{1}{2}\pi d$$
$$\approx 2(12\text{ m}) + 4\text{ m} + \frac{1}{2}(3.14)(4\text{ m})$$
$$= 34.28\text{ m}$$

• **L = 12 m, W = 4 m, d = 4 m. *Note:* The diameter of the circle is equal to the width of the rectangle.**

The perimeter is approximately 34.28 m.

EXAMPLE 4

Find the perimeter of the composite figure. Use $\frac{22}{7}$ for π.



Solution

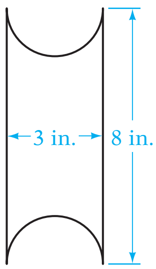
Perimeter of composite figure = sum of lengths of the four sides + $\frac{1}{2}$ the circumference of the circle

$$P = 4s + \frac{1}{2}\pi d$$
$$\approx 4(5\text{ cm}) + \frac{1}{2}\left(\frac{22}{7}\right)(7\text{ cm})$$
$$= 20\text{ cm} + 11\text{ cm} = 31\text{ cm}$$

The perimeter is approximately 31 cm.

YOU TRY IT 4

Find the perimeter of the composite figure. Use 3.14 for π.



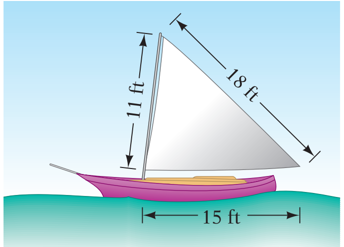
Your solution

Solution on p. S29

OBJECTIVE C *To solve application problems*

EXAMPLE 5

The dimensions of a triangular sail are 18 ft, 11 ft, and 15 ft. What is the perimeter of the sail?



Strategy

To find the perimeter, use the formula for the perimeter of a triangle.

Solution

$P = a + b + c$

$= 18 \text{ ft} + 11 \text{ ft} + 15 \text{ ft} = 44 \text{ ft}$

The perimeter of the sail is 44 ft.

YOU TRY IT 5

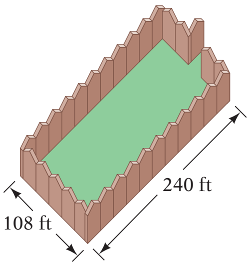
What is the perimeter of a standard piece of computer paper that measures $8\frac{1}{2}$ in. by 11 in.?

Your strategy

Your solution

EXAMPLE 6

If fencing costs \$6.75 per foot, how much will it cost to fence a rectangular lot that is 108 ft wide and 240 ft long?



Strategy

To find the cost of the fence:

- Find the perimeter of the lot.
- Multiply the perimeter by the per-foot cost of the fencing.

Solution

$P = 2L + 2W$

$= 2(240 \text{ ft}) + 2(108 \text{ ft})$

$= 480 \text{ ft} + 216 \text{ ft} = 696 \text{ ft}$

$\text{Cost} = 696 \times 6.75 = 4698$

The cost to fence the lot is \$4698.

YOU TRY IT 6

Metal stripping is being installed around a workbench that is 0.74 m wide and 3 m long. At \$4.49 per meter, find the cost of the metal stripping. Round to the nearest cent.

Your strategy

Your solution

Solutions on p. S29

12.2 EXERCISES

Concept Check

1. Complete each sentence with the name of a polygon or a number of sides.

a. A ______________ has three sides.

b. A polygon with six sides is called a ______________.

c. A polygon with four equal sides is called a ______________. If the polygon also has four equal angles, then it is called a ______________.

d. An isosceles triangle has ______________ sides of equal length.

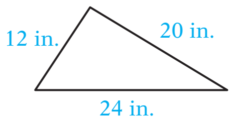
2. What is the perimeter of a circle called?

3. What is the formula for the perimeter of a rectangle?

4. What is the formula for the circumference of a circle?

OBJECTIVE A *To find the perimeter of plane geometric figures*

For Exercises 5 to 12, find the perimeter or circumference of the given figure. Use 3.14 for π.

5.


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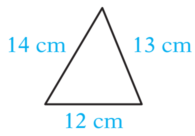

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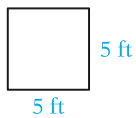

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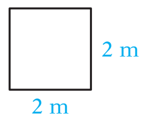

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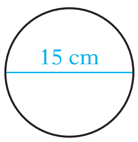

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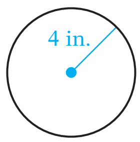

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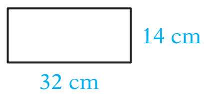


12.


13. Find the perimeter of a triangle with sides of lengths 2 ft 4 in., 3 ft, and 4 ft 6 in.

14. Find the perimeter of a rectangle with a length of 2 m and a width of 0.8 m.

15. Find the circumference of a circle with a radius of 8 cm. Use 3.14 for π.

16. Find the circumference of a circle with a diameter of 14 in. Use $\frac{22}{7}$ for π.

17. Find the perimeter of a square in which each side is 60 m.

18. Find the perimeter of a triangle in which each side is $1\frac{2}{3}$ ft.

19. Farming A horse trainer wants to build a rectangular corral that is 60 ft wide and 75 ft long. How many feet of fencing will the trainer need to build the corral?

20. Money The diameter of a quarter dollar is 24.26 mm. What is the circumference of a quarter dollar? Use 3.14 for π. Round to the nearest hundredth.

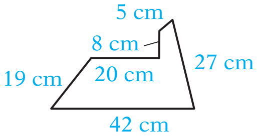
21. The length of a side of a square is equal to the diameter of a circle. Which is greater, the perimeter of the square or the circumference of the circle?

22. The length of a rectangle is equal to the diameter of a circle, and the width of the rectangle is equal to the radius of the same circle. Which is greater, the perimeter of the rectangle or the circumference of the circle?

OBJECTIVE B *To find the perimeter of composite geometric figures*

For Exercises 23 to 30, find the perimeter. Use 3.14 for π.

23.



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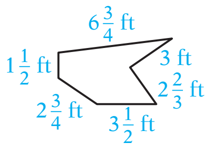

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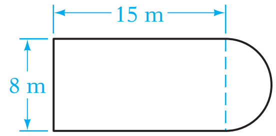

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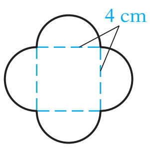

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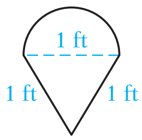

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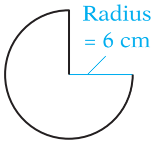

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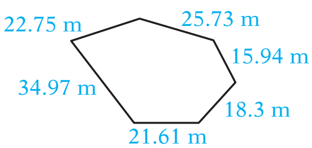
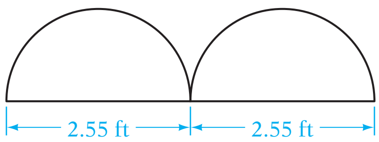
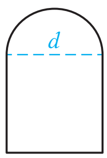


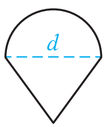
30.



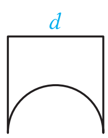
For Exercises 31 and 32, determine whether the perimeter of the given composite figure is less than, equal to, or greater than the perimeter of the figure shown at the right. The figure at the right is made up of a square and one-half a circle of diameter *d*.



31. A figure formed by an equilateral triangle and one-half a circle of diameter *d*



32. A figure formed by a square and one-half a circle of diameter *d*



OBJECTIVE C *To solve application problems*

33. Landscaping How many feet of fencing should be purchased to enclose a rectangular garden that is 18 ft long and 12 ft wide?

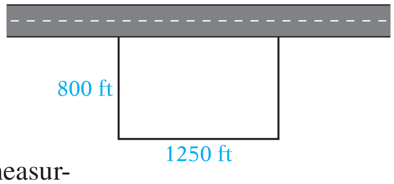
34. Interior Design Wall-to-wall carpeting is installed in a room that is 12 ft long and 10 ft wide. The edges of the carpet are nailed to the floor. Along how many feet must the carpet be nailed down?

35. Quilting How many feet of binding are required to bind the edge of a rectangular quilt that measures 3.5 ft by 8.5 ft?

36. Carpentry Find the length of molding needed to trim a circular table that is 3.8 ft in diameter. Use 3.14 for π.

37. Race Tracks The first circular dog race track opened in 1919 in Emeryville, California. The radius of the circular track was 157.64 ft. Find the circumference of the track. Use 3.14 for π. Round to the nearest whole number.

38. Landscaping The rectangular lot shown in the figure at the right is being fenced. The fencing along the road costs \$6.20 per foot. The rest of the fencing costs \$5.85 per foot. Find the total cost to fence the lot.

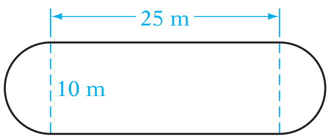


39. Sewing Bias binding is to be sewed around the edge of a rectangular quilt measuring 72 in. by 45 in. Each package of bias binding costs \$5.50 and contains 15 ft of binding. How many packages of bias binding are needed for the quilt?

40. Travel A bicycle tire has a diameter of 24 in. How many feet does the bicycle travel when the wheel makes five revolutions? Use 3.14 for π.

41. Travel A tricycle tire has a diameter of 12 in. How many feet does the tricycle travel when the wheel makes eight revolutions? Use 3.14 for π.

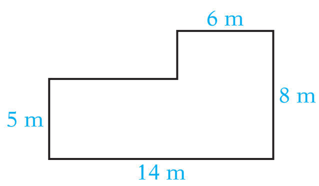
Architecture For Exercises 42 and 43, use the floor plan of a roller rink shown at the right.



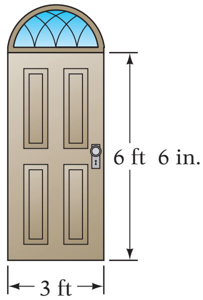
42. Use estimation to determine whether the perimeter is less than 70 m or greater than 70 m.

43. Calculate the perimeter of the roller rink. Use 3.14 for π.

44. Home Improvement A rain gutter is being installed on a home that has the dimensions shown in the figure at the right. At a cost of \$11.30 per meter, how much will it cost to install the rain gutter?



45. **Home Improvement** Find the length of weather stripping installed around the arched door shown in the figure at the right. Use 3.14 for π.



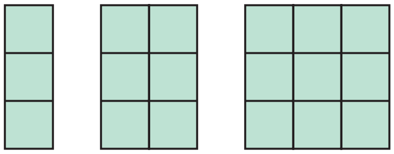
46. **Astronomy** The distance from Earth to the sun is 93,000,000 mi. Approximate the distance Earth travels in making one revolution about the sun. Use 3.14 for π.

Critical Thinking

47. **a.** If the diameter of a circle is doubled, how many times larger is the resulting circumference?

b. If the radius of a circle is doubled, how many times larger is the resulting circumference?

48. **Geometry** In the pattern below, the length of one side of a square is 1 unit. Find the perimeter of the eighth figure in the pattern.



49. **Forestry** A forest ranger must determine the diameter of a redwood tree. Explain how the ranger could do this without cutting down the tree.

Projects or Group Activities

50. **Fractals** The diagrams below show the first three stages of the Sierpinski Triangle, a pattern in Italian mosaics that dates from the 13th century. The pattern is named for the Polish mathematician Waclaw Sierpinski, who studied the design and its properties in the early part of the 20th century.

To create the design, first place an equilateral triangle inside another equilateral triangle, as shown in Figure A. Then place three more equilateral triangles inside the unshaded triangles of Figure A, forming Figure B. Repeat this process again to make Figure C from Figure B. The process can be repeated indefinitely.

Determine the sum of the perimeters of all the shaded triangles in Figure C.

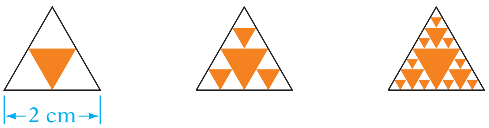


Figure A Figure B Figure C

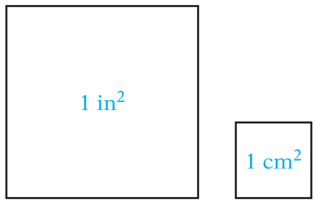
SECTION

12.3 Area

OBJECTIVE A *To find the area of geometric figures*

Area is a measure of the amount of surface in a region. Area can be used to describe, for example, the size of a rug, a parking lot, a farm, or a national park. Area is measured in square units.

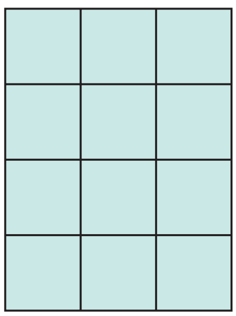
A square that measures 1 in. on each side has an area of 1 square inch, which is written 1 in^2.



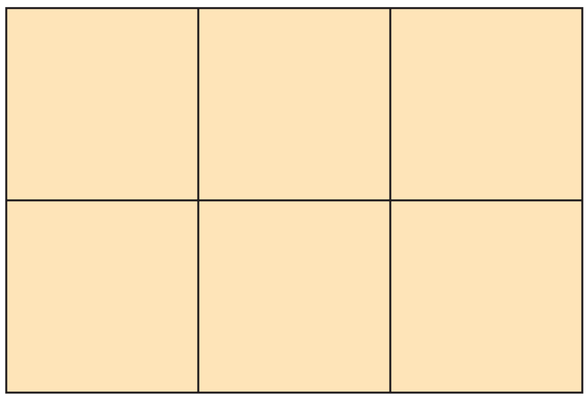
A square that measures 1 cm on each side has an area of 1 square centimeter, which is written 1 cm^2.

Larger areas can be measured in square feet (ft^2), square meters (m^2), square miles (mi^2), acres (43,560 ft^2), or any other square unit.

The area of a geometric figure is the number of squares that are necessary to cover the figure. In the figures below, two rectangles have been drawn and covered with squares. In the figure on the left, 12 squares, each of area 1 cm^2, were used to cover the rectangle. The area of the rectangle is 12 cm^2. In the figure on the right, 6 squares, each of area 1 in^2, were used to cover the rectangle. The area of the rectangle is 6 in^2.



The area of the rectangle is 12 cm^2.



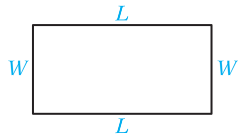
The area of the rectangle is 6 in^2.

Note from the above figures that the area of a rectangle can be found by multiplying the length of the rectangle by its width.

Area of a Rectangle

Let L be the length of a rectangle and W be the width of a rectangle.

The area of the rectangle is $A = LW$.



EXAMPLE

Find the area of the rectangle shown at the right.

$A = LW$

$= (8\text{ ft})(5\text{ ft})$ • $L = 8$ ft, $W = 5$ ft

$= 40\text{ ft}^2$

The area of the rectangle is 40 ft^2.

APPLY THE CONCEPT

A carpet installer charges $.30 per square foot to install wall-to-wall carpeting. How much would the installer charge to install wall-to-wall carpeting in a rectangular room that measures 12 ft by 14 ft?

To find the cost, first find the area of the room. Then multiply the area by the cost per square foot.

$A = LW$

$= (14\text{ ft})(12\text{ ft})$ • $L = 14$ ft, $W = 12$ ft

$= 168\text{ ft}^2$

The area of the room is 168 ft^2.

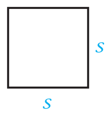
Cost $= 168 \cdot 0.30 = 50.40$

The installer would charge $50.40 to install the carpet.

Area of a Square

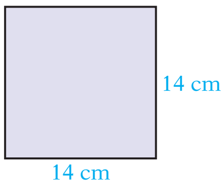
Let s be the length of one side of a square.

The area of the square is $A = s^2$.



EXAMPLE

Find the area of the square shown at the right.



$A = s^2$

$= (14\text{ cm})^2$ • $s = 14$ cm

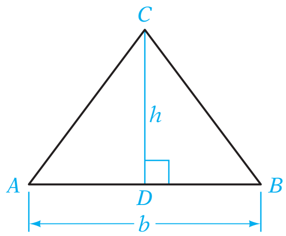
$= 196\text{ cm}^2$

The area of the square is 196 cm^2.

Area of a Triangle

In the figure at the right, $\overline{AB}$ is the base b of the triangle, and $\overline{CD}$, which is perpendicular to the base b, is the height h.

The area of a triangle is $A = \frac{1}{2}bh$.



EXAMPLE

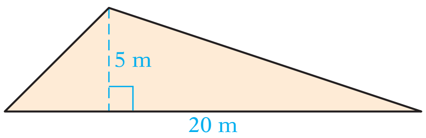
Find the area of the triangle shown at the right.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(20 \text{ m})(5 \text{ m})$ • $b = 20$ m, $h = 5$ m

$= 50 \text{ m}^2$

The area of the triangle is 50 m^2.



Integrating Technology

To calculate the area of the triangle shown in the Example, you can enter

20 × 5 ÷ 2 =

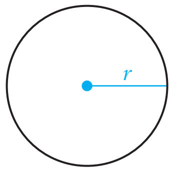
or

.5 × 20 × 5 =

Area of a Circle

Let r be the radius of a circle.

The area of the circle is $A = \pi r^2$.



EXAMPLE

Find the area of the circle shown at the right.

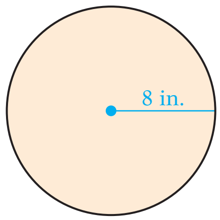
$A = \pi r^2$

$= \pi(8 \text{ in.})^2 = 64\pi \text{ in}^2$

$\approx 64 \cdot 3.14 \text{ in}^2 = 200.96 \text{ in}^2$

The area is exactly 64π in^2.

The area is approximately 200.96 in^2.



Take Note

Note that we gave two answers for the area of the circle. The *exact* answer, which includes the symbol for π, and an *approximate* answer, which uses an approximation for π.

APPLY THE CONCEPT

The bottom of a circular children's wading pool has a diameter of 5 ft. What is the area of the bottom of the pool?

To find the area, first find the radius of the bottom of the pool. Then use the equation $A = \pi r^2$ to find the area. Use 3.14 for π.

$$r = \frac{\text{diameter}}{2} = \frac{5 \text{ ft}}{2} = 2.5 \text{ ft}$$

$$A = \pi r^2$$
$$= \pi(2.5 \text{ ft})^2$$
$$\approx 3.14(2.5 \text{ ft})^2$$
$$= 19.625 \text{ ft}^2$$

The area of the bottom of the pool is 19.625 ft^2.

EXAMPLE 1

Find the area of a circle with a diameter of 9 cm. Use 3.14 for π.

Solution

$$r = \frac{1}{2}d = \frac{1}{2}(9 \text{ cm}) = 4.5 \text{ cm}$$
$$A = \pi r^2$$
$$\approx 3.14(4.5 \text{ cm})^2 = 63.585 \text{ cm}^2$$

The area is approximately 63.585 cm^2.

YOU TRY IT 1

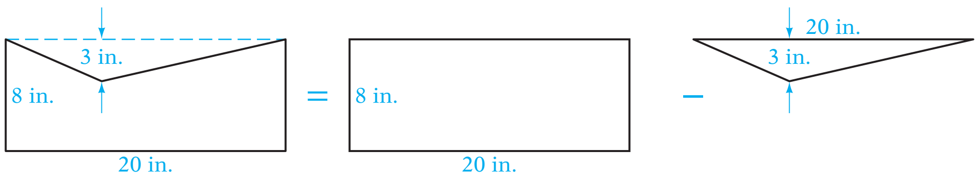
Find the area of a triangle with a base of 24 in. and a height of 14 in.

Your solution

Solution on p. S29

OBJECTIVE B *To find the area of composite geometric figures*

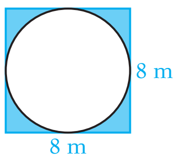
The area of the composite figure shown below is found by calculating the area of the rectangle and then subtracting the area of the triangle.



$$A = LW - \frac{1}{2}bh$$
$$= (20 \text{ in.})(8 \text{ in.}) - \frac{1}{2}(20 \text{ in.})(3 \text{ in.}) = 160 \text{ in}^2 - 30 \text{ in}^2 = 130 \text{ in}^2$$

EXAMPLE 2

Find the area of the shaded portion of the figure. Use 3.14 for π.



Solution

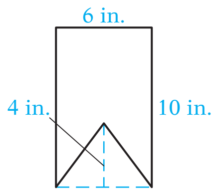
Area of shaded portion = area of square − area of circle

$$A = s^2 - \pi r^2$$
$$= (8 \text{ m})^2 - \pi(4 \text{ m})^2$$
$$\approx 64 \text{ m}^2 - 3.14(16 \text{ m}^2)$$
$$= 64 \text{ m}^2 - 50.24 \text{ m}^2 = 13.76 \text{ m}^2$$

The area is approximately 13.76 m^2.

YOU TRY IT 2

Find the area of the composite figure.



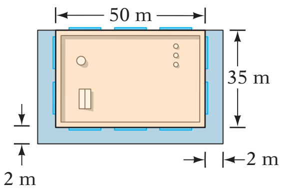
Your solution

Solution on p. S29

OBJECTIVE C *To solve application problems*

EXAMPLE 3

A walkway 2 m wide is built along the front and along both sides of a building, as shown in the figure. Find the area of the walkway.



Strategy

To find the area of the walkway, add the area of the front section (54 m · 2 m) and the area of the two side sections (each 35 m · 2 m).

Solution

Area of walkway = area of front section + 2(area of one side section)

$$A = (54 \text{ m})(2 \text{ m}) + 2(35 \text{ m})(2 \text{ m})$$
$$= 108 \text{ m}^2 + 140 \text{ m}^2$$
$$= 248 \text{ m}^2$$

The area of the walkway is 248 m^2.

YOU TRY IT 3

New carpet is installed in a room measuring 9 ft by 12 ft. Find the area of the room in square yards. ($9 \text{ ft}^2 = 1 \text{ yd}^2$)

Your strategy

Your solution

Solution on p. S30

12.3 EXERCISES

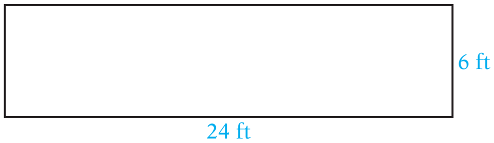
Concept Check

1. Give the formula for the area of each figure:
 a. Rectangle
 b. Circle

2. State whether each unit could be used to measure *perimeter* or *area*.
 a. mm
 b. ft^2
 c. mi
 d. km
 e. yd^2

OBJECTIVE A *To find the area of geometric figures*

For Exercises 3 to 10, find the area of the given figure. Use 3.14 for π.

3.


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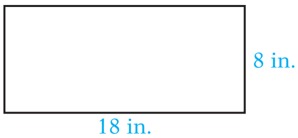

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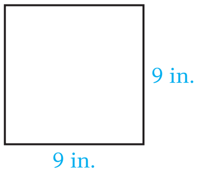

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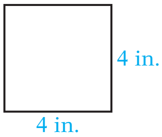
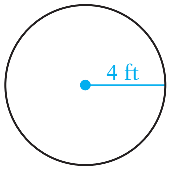
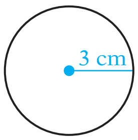
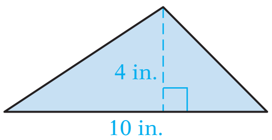
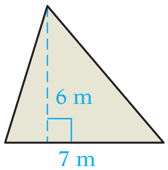


7.


8.


9.


10.


11. Find the area of a right triangle with a base of 3 cm and a height of 1.42 cm.

12. Find the area of a triangle with a base of 3 ft and a height of $\frac{2}{3}$ ft.

13. Find the area of a square with a side of 4 ft.

14. Find the area of a square with a side of 10 cm.

15. Find the area of a rectangle with a length of 43 in. and a width of 19 in.

16. Find the area of a rectangle with a length of 82 cm and a width of 20 cm.

17. Find the area of a circle with a radius of 7 in. Use $\frac{22}{7}$ for π.

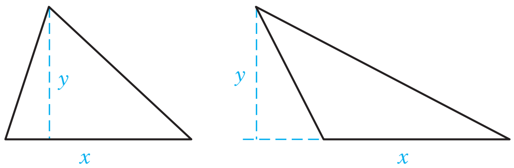
18. Find the area of a circle with a diameter of 40 cm. Use 3.14 for π.

19. **Sports** A square is formed by connecting the bases of a professional baseball field. The length of each side of the square is 90 ft. What is the area of the baseball field?

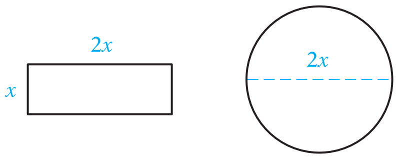
20. **Camping** The back of a tent is in the shape of a triangle with a base of 6 ft and a height of 5 ft. What is the area of the back of the tent?

For Exercises 21 and 22, determine whether the area of the first figure is less than, equal to, or greater than the area of the second figure.

21.



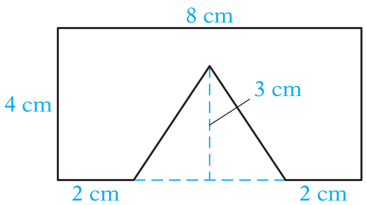
22.



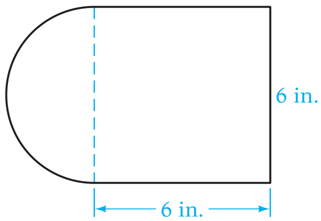
OBJECTIVE B *To find the area of composite geometric figures*

For Exercises 23 to 30, find the area. Use 3.14 for π.

23.



24.



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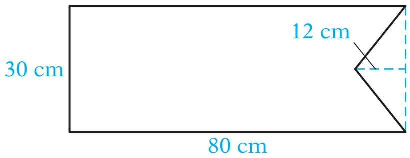

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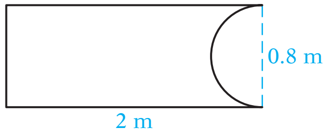
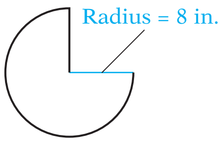


27.



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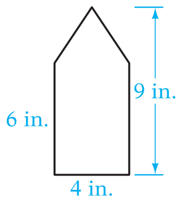
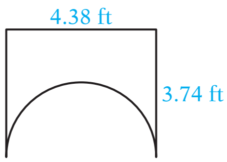
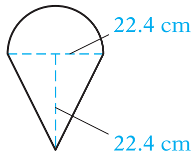


29.



30.



31. Determine whether the area of Figure 1 below is less than, equal to, or greater than the area of Figure 2.

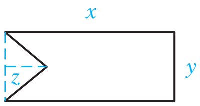


Figure 1

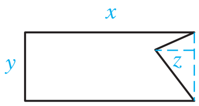


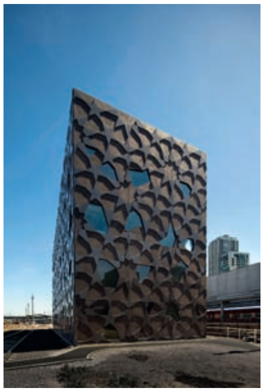
Figure 2

OBJECTIVE C *To solve application problems*

32. **Architecture** See the news clipping at the right. Use the dimensions given for the Yardmaster Building to estimate the cost of carpeting the two floors of office space if the cost of the carpet is \$18 per square meter.

33. **Sports** Artificial turf is being used to cover a playing field. The field is rectangular with a length of 100 yd and a width of 75 yd. How much artificial turf must be purchased to cover the field?

34. **Telescopes** The telescope lens of the Hale telescope at Mount Palomar, California, has a diameter of 200 in. Find the area of the lens. Use 3.14 for π.



Courtesy of John Gollings Photography and McBride Charles Ryan, Architects

In the NEWS!

Railroad Building Stands Out

The new Yardmaster Building, home to railroad operation facilities in Melborne, Australia, makes a striking contrast to the surrounding railroad tracks. The unusual four-story rectangular building is about 30 m long and 10 m wide. Two of its four stories are devoted to office space.

Source: www.designboom.com

35. **Home Improvement** You plan to stain the wooden deck attached to your house. The deck measures 10 ft by 8 ft. A quart of stain will cost \$11.87 and will cover 50 ft^2. How many quarts of stain should you buy?

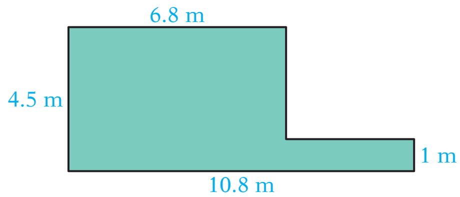
36. **Interior Design** A fabric wall hanging is to fill a space that measures 5 m by 3.5 m. Allowing for 0.1 m of the fabric to be folded back along each edge, how much fabric must be purchased for the wall hanging?

37. **Agriculture** An irrigation system waters a circular field that has a 50-foot radius. Find the area watered by the irrigation system. Use 3.14 for π.

© Corbis Premium RF/Alamy

38. **Geography** The shape of the state of Massachusetts can be approximated by a rectangle with a length of 150 mi and a width of 70 mi. Use these dimensions to approximate the area of Massachusetts.

Interior Design A carpet is to be installed in one room and a hallway, as shown in the diagram at the right. For Exercises 39 to 42, state whether the given expression can be used to calculate the area of the carpet in square meters.



39. 4.5(6.8) + 10.8(1)

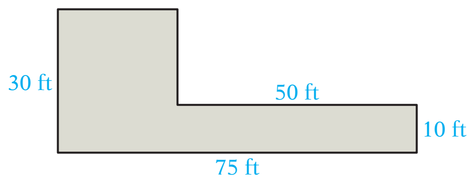
40. 4.5(10.8) – 3.5(4)

41. 10.8(1) + 3.5(6.8)

42. 4.5(6.8) + 1(4)

43. **Interior Design** Use the diagram for Exercises 39 to 42. At a cost of \$28.50 per square meter, how much will it cost to carpet the area?

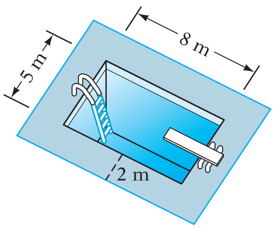
44. **Landscaping** Find the area of a concrete driveway with the measurements shown in the figure.



45. **Interior Design** You want to tile your kitchen floor. The floor measures 12 ft by 9 ft. How many tiles, each a square with side $1\frac{1}{2}$ ft, should you purchase for the job?

46. **Interior Design** You are wallpapering two walls of a child's room. One wall measures 9 ft by 8 ft, and the other measures 11 ft by 8 ft. The wallpaper costs \$34.50 per roll, and each roll of the wallpaper will cover 40 ft^2. What is the cost to wallpaper the two walls?

47. Construction Find the area of the 2-meter boundary around the swimming pool shown in the figure.

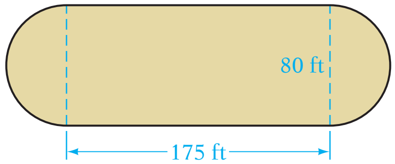


48. Parks An urban renewal project involves reseeding a park that is in the shape of a square, 60 ft on each side. Each bag of grass seed costs \$5.75 and will seed 1200 ft^2. How much money should be budgeted for buying grass seed for the park?

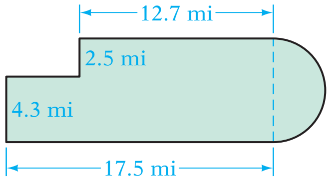
49. Architecture The roller rink shown in the figure at the right is to be covered with hardwood flooring.

a. Without doing the calculations, indicate whether the area of the rink is more than 8000 ft^2 or less than 8000 ft^2.

b. Calculate how much hardwood flooring is needed to cover the roller rink. Use 3.14 for π.



50. Parks Find the total area of the national park with the dimensions shown in the figure at the right. Use 3.14 for π.



51. Interior Design Find the cost of plastering the walls of a room that is 22 ft wide, 25 ft 6 in. long, and 8 ft high. Subtract 120 ft^2 for windows and doors. The cost is \$3 per square foot.

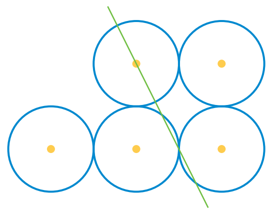
Critical Thinking

52. a. If both the length and the width of a rectangle are doubled, how many times larger is the area of the resulting rectangle?

b. If the radius of a circle is doubled, what happens to the area?

c. If the diameter of a circle is doubled, what happens to the area?

53. The circles at the right are identical. Is the area in the circles to the left of the line equal to, less than, or greater than the area in the circles to the right of the line? Explain your answer.



54. Determine whether the statement is always true, sometimes true, or never true.

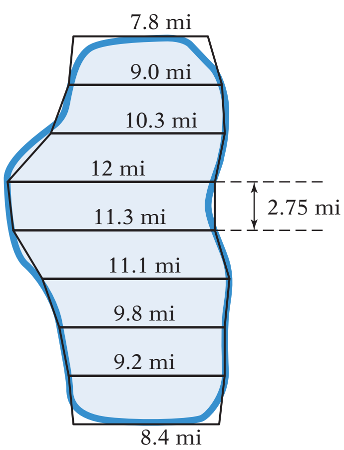
a. If two triangles have the same perimeter, then the triangles have the same area.

b. If two rectangles have the same area, then the rectangles have the same perimeter.

c. If two squares have the same area, then the sides of the squares have the same length.

Projects or Group Activities

55. Lake Tahoe One way to measure the area of an irregular figure, such as a lake, is to divide the area into trapezoids that have the same height. Then measure the length of each base, calculate the area of each trapezoid, and add the areas. The figure at the right gives approximate dimensions for Lake Tahoe, which straddles the California and Nevada borders. Approximate the area of Lake Tahoe using the given trapezoids. Round to the nearest tenth. *Note:* The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of the bases.

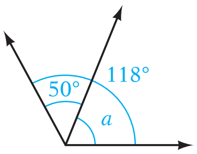


CHECK YOUR PROGRESS: CHAPTER 12

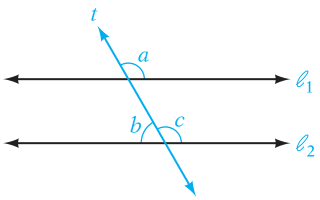
1. Given that $MN = 20$, $NO = 24$, and $MP = 72$, find OP.

2. Suppose $\angle A = 27°$. What is the complement of $\angle A$? What is the supplement of $\angle A$?

3. Find the measure of $\angle a$.



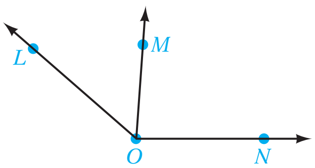
4. In the figure, $l_1 \parallel l_2$ and $\angle c = 120°$. Find the measures of $\angle a$ and $\angle b$.



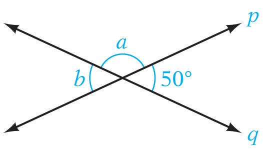
5. One angle of a right triangle measures 22°. What are the measures of the other two angles?

6. Two angles of a triangle measure 54° and 112°. What is the measure of the third angle?

7. In the figure, $\angle LON = 139°$ and $\angle MON = 86°$. Find the measure of $\angle LOM$.



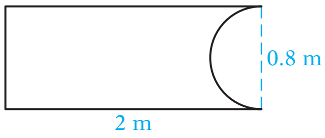
8. Find the measures of $\angle a$ and $\angle b$.



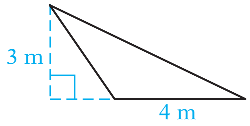
9. Find the perimeter of a rectangle with a length of 3.25 m and a width of 75 cm.

10. Find the circumference of a circle whose radius is 3.6 in. Use 3.14 for π.

11. Find the perimeter of the figure. Use 3.14 for π.

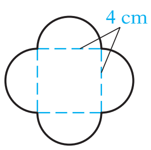


12. Find the area of the triangle.



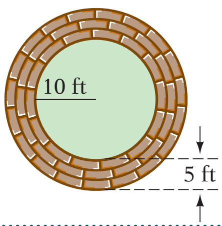
13. Find the area of a circle whose diameter is 6 in. Use 3.14 for π.

14. Find the area of the figure. Use 3.14 for π.



15. **Carpeting** Wall-to-wall carpeting is being installed in a room that is 12 ft wide and 14 ft long. If the carpet costs \$3.25 per square foot, what is the cost of the carpet?

16. **Landscaping** A circular brick walkway surrounds a garden as shown in the diagram at the right. What is the area of the brick walkway? Use 3.14 for π.



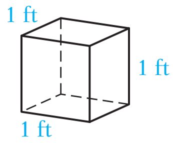
SECTION

12.4 Volume

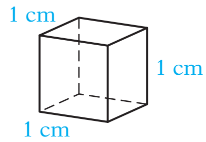
OBJECTIVE A *To find the volume of geometric solids*

Volume is a measure of the amount of space inside a closed surface, or figure in space. Volume can be used to describe, for example, the amount of heating gas used for cooking, the amount of concrete delivered for the foundation of a house, or the amount of water in storage for a city's water supply.

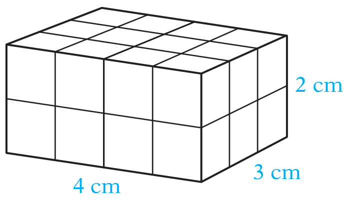
A cube that is 1 ft on each side has a volume of 1 cubic foot, which is written 1 ft^3.



A cube that measures 1 cm on each side has a volume of 1 cubic centimeter, which is written 1 cm^3.



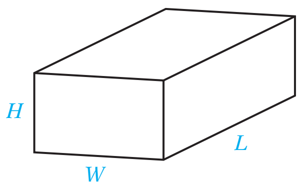
The volume of a solid is the number of cubes that are necessary to fill the solid exactly. The volume of the rectangular solid at the right is 24 cm^3 because it will hold exactly 24 cubes, each 1 cm on a side. Note that the volume can be found by multiplying the length times the width times the height.



Volume of a Rectangular Solid

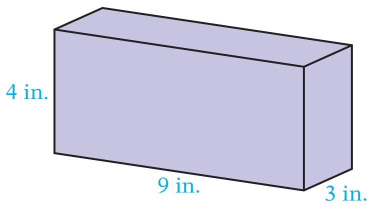
Let L be the length, W be the width, and H be the height of a rectangular solid.

The volume of the rectangular is $V = LWH$.



EXAMPLE

Find the volume of the rectangular solid shown at the right.



$V = LWH$

$= (9 \text{ in.})(3 \text{ in.})(4 \text{ in.})$ • $L = 9$ **in.**, $W = 3$ **in.**, $H = 4$ **in.**

$= 108 \text{ in}^3$

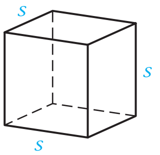
The volume of the rectangular solid is 108 in^3.

A **cube** is a rectangular solid for which the length, width, and height are all equal. The volume of a cube is found by multiplying the length of a side of the cube times itself three times ("side cubed").

Volume of a Cube

Let s be the length of one side of a cube.

The volume of the cube is $A = s^3$.



EXAMPLE

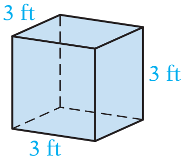
Find the volume of the cube shown at the right.

$A = s^3$

$= (3\text{ ft})^3$ • $s = 3$ ft

$= 27\text{ ft}^3$

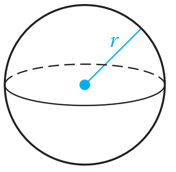
The volume of the cube is 27 ft^3.



Volume of a Sphere

Let r be the radius of a sphere.

The volume of the sphere is $V = \frac{4}{3}\pi r^3$.



EXAMPLE

Find the volume of the sphere shown at the right. Use 3.14 for π. Round to the nearest hundredth.

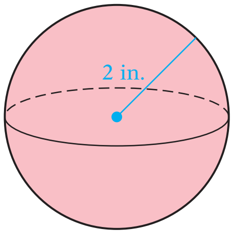
$V = \frac{4}{3}\pi r^3$

$\approx \frac{4}{3}(3.14)(2\text{ in.})^3$ • $r = 2$ in.

$= \frac{4}{3}(3.14)(8\text{ in}^3)$

$\approx 33.49\text{ in}^3$

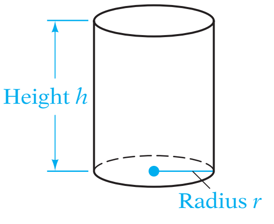
The volume is approximately 33.49 in^3.



Volume of a Cylinder

Let r be the radius of a cylinder.

The volume of the cylinder is $V = \pi r^2 h$.



EXAMPLE

Find the volume of the cylinder shown below. Use 3.14 for π. Round to the nearest hundredth.

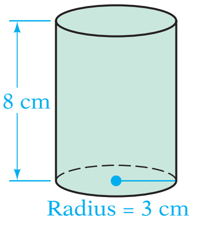
$V = \pi r^2 h$

$\approx 3.14(3\text{ cm})^2(8\text{ cm})$ • $r = 3$ cm, $h = 8$ cm

$= 3.14(9\text{ cm}^2)(8\text{ cm})$

$= 226.08\text{ cm}^3$

The volume of the cylinder is approximately 226.08 cm^3.



EXAMPLE 1

Find the volume of a rectangular solid with a length of 3 ft, a width of 1.5 ft, and a height of 2 ft.

Solution

$V = LWH$
$= (3 \text{ ft})(1.5 \text{ ft})(2 \text{ ft})$
$= 9 \text{ ft}^3$

The volume is 9 ft^3.

YOU TRY IT 1

Find the volume of a rectangular solid with a length of 8 cm, a width of 3.5 cm, and a height of 4 cm.

Your solution

EXAMPLE 2

Find the volume of a cube that has a side measuring 2.5 in.

Solution

$V = s^3$
$= (2.5 \text{ in.})^3$
$= 15.625 \text{ in}^3$

The volume is 15.625 in^3.

YOU TRY IT 2

Find the volume of a cube with a side of length 5 cm.

Your solution

EXAMPLE 3

Find the volume of a cylinder with a radius of 12 cm and a height of 65 cm. Use 3.14 for π.

Solution

$V = \pi r^2 h$
$\approx 3.14(12 \text{ cm})^2(65 \text{ cm})$
$= 3.14(144 \text{ cm}^2)(65 \text{ cm})$
$= 29{,}390.4 \text{ cm}^3$

The volume is approximately $29{,}390.4 \text{ cm}^3$.

YOU TRY IT 3

Find the volume of a cylinder with a diameter of 14 in. and a height of 15 in. Use $\frac{22}{7}$ for π.

Your solution

Solutions on p. S30

EXAMPLE 4

Find the volume of a sphere with a diameter of 12 in. Use 3.14 for π.

Solution

$r = \frac{1}{2}d = \frac{1}{2}(12 \text{ in.}) = 6 \text{ in.}$ • **Find the radius.**

$V = \frac{4}{3}\pi r^3$ • **Use the formula for the volume of a sphere.**

$\approx \frac{4}{3}(3.14)(6 \text{ in.})^3$

$= \frac{4}{3}(3.14)(216 \text{ in}^3)$

$= 904.32 \text{ in}^3$

The volume is approximately 904.32 in^3.

YOU TRY IT 4

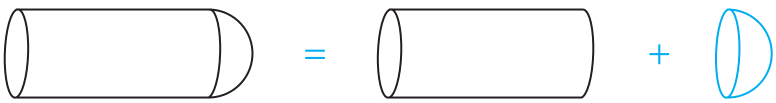
Find the volume of a sphere with a radius of 3 m. Use 3.14 for π.

Your solution

Solution on p. S30

OBJECTIVE B *To find the volume of composite geometric solids*

A **composite geometric solid** is a solid made from two or more geometric solids. The solid shown is made from a cylinder and one-half of a sphere.



Volume of the composite solid $=$ volume of the cylinder $+ \frac{1}{2}$ the volume of the sphere

HOW TO 1 Find the volume of the composite solid shown above if the radius of the base of the cylinder is 3 in. and the height of the cylinder is 10 in. Use 3.14 for π.

The volume equals the volume of a cylinder plus one-half the volume of a sphere. The radius of the sphere equals the radius of the base of the cylinder.

$$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$\approx 3.14(3 \text{ in.})^2(10 \text{ in.}) + \frac{1}{2}\left(\frac{4}{3}\right)(3.14)(3 \text{ in.})^3$$

$$= 3.14(9 \text{ in}^2)(10 \text{ in.}) + \frac{1}{2}\left(\frac{4}{3}\right)(3.14)(27 \text{ in}^3)$$

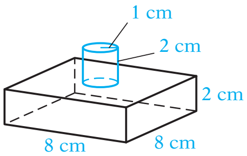
$$= 282.6 \text{ in}^3 + 56.52 \text{ in}^3$$

$$= 339.12 \text{ in}^3$$

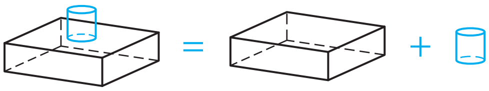
The volume is approximately 339.12 in^3.

EXAMPLE 5

Find the volume of the solid shown in the figure. Use 3.14 for π.



Solution



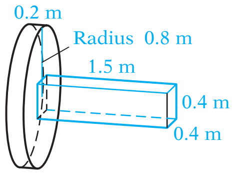
Volume of solid = volume of rectangular solid + volume of cylinder

$$V = LWH + \pi r^2 h$$
$$\approx (8\text{ cm})(8\text{ cm})(2\text{ cm}) + 3.14(1\text{ cm})^2(2\text{ cm})$$
$$= 128\text{ cm}^3 + 6.28\text{ cm}^3$$
$$= 134.28\text{ cm}^3$$

The volume is approximately 134.28 cm^3.

YOU TRY IT 5

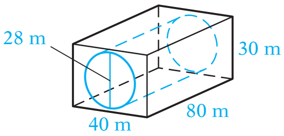
Find the volume of the solid shown in the figure. Use 3.14 for π.



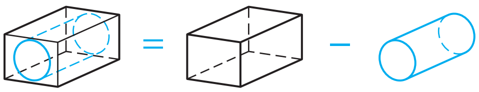
Your solution

EXAMPLE 6

Find the volume of the solid shown in the figure. Use 3.14 for π.



Solution



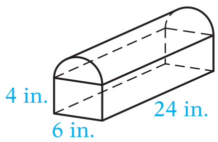
Volume of solid = volume of rectangular solid − volume of cylinder

$$V = LWH - \pi r^2 h$$
$$\approx (80\text{ m})(40\text{ m})(30\text{ m}) - 3.14(14\text{ m})^2(80\text{ m})$$
$$= 96{,}000\text{ m}^3 - 49{,}235.2\text{ m}^3$$
$$= 46{,}764.8\text{ m}^3$$

The volume is approximately 46,764.8 m^3.

YOU TRY IT 6

Find the volume of the solid shown in the figure. Use 3.14 for π.



Your solution

Solutions on p. S30

OBJECTIVE C *To solve application problems*

EXAMPLE 7

An aquarium is 28 in. long, 14 in. wide, and 16 in. high. Find the volume of the aquarium.

Strategy

To find the volume of the aquarium, use the formula for the volume of a rectangular solid.

Solution

$$V = LWH$$
$$= (28\text{ in.})(14\text{ in.})(16\text{ in.})$$
$$= 6272\text{ in}^3$$

The volume of the aquarium is 6272 in^3.

YOU TRY IT 7

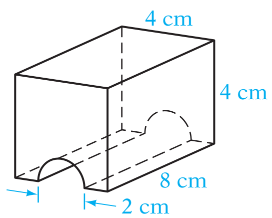
Find the volume of a freezer that is 7 ft long, 3 ft high, and 2.5 ft wide.

Your strategy

Your solution

EXAMPLE 8

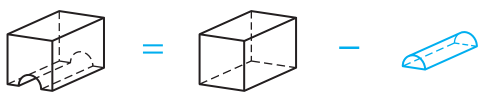
Find the volume of the bushing shown in the figure below. Use 3.14 for π.



Strategy

To find the volume of the bushing, subtract the volume of the half-cylinder from the volume of the rectangular solid.

Solution



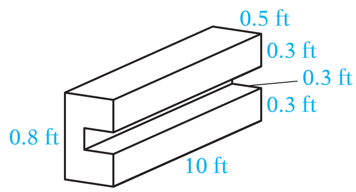
Volume of bushing = volume of rectangular solid − $\frac{1}{2}$ volume of cylinder

$$V = LWH - \frac{1}{2}\pi r^2 h$$
$$\approx (8\text{ cm})(4\text{ cm})(4\text{ cm}) - \frac{1}{2}(3.14)(1\text{ cm})^2(8\text{ cm})$$
$$= 128\text{ cm}^3 - 12.56\text{ cm}^3$$
$$= 115.44\text{ cm}^3$$

The volume of the bushing is approximately 115.44 cm^3.

YOU TRY IT 8

Find the volume of the channel iron shown in the figure below.



Your strategy

Your solution

Solutions on p. S30

12.4 EXERCISES

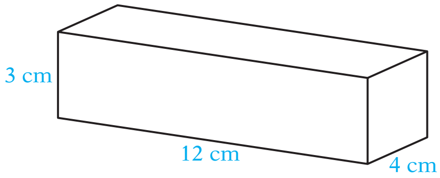
Concept Check

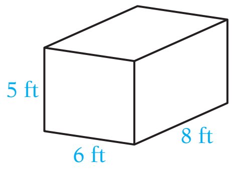
1. Give the formula for the volume of each solid.
a. Cube **b.** Sphere **c.** Cylinder

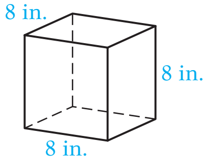
2. State whether each unit could be used to measure perimeter, area, or volume.
a. km **b.** yd^3 **c.** cm^3 **d.** mi^2
e. mm^3 **f.** in. **g.** dm **h.** hm^2

OBJECTIVE A *To find the volume of geometric solids*

For Exercises 3 to 10, find the volume. If necessary, round to the nearest hundredth. Use 3.14 for π.

3.


4.


5.


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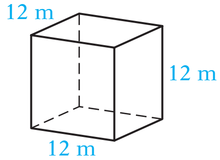

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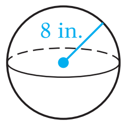

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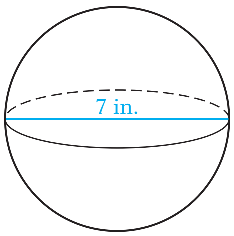
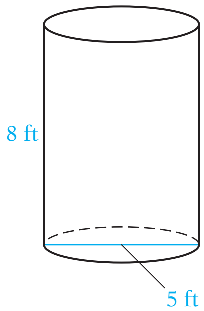


9.


10.


11. Find the volume, in cubic meters, of a rectangular solid with a length of 2 m, a width of 80 cm, and a height of 4 m.

12. Find the volume, in cubic meters, of a rectangular solid with a length of 1.15 m, a width of 60 cm, and a height of 25 cm.

13. Find the volume of a sphere with an 11-millimeter radius. Use 3.14 for π. Round to the nearest hundredth.

14. Find the volume of a cube with a side of length 2.14 m. Round to the nearest tenth.

15. Find the volume of a cylinder with a diameter of 12 ft and a height of 30 ft. Use 3.14 for π.

16. Find the volume of a sphere with a 6-foot diameter. Use 3.14 for π.

17. Find the volume of a cube with a side of length $3\frac{1}{2}$ ft.

18. Find the volume of a cylinder with a radius of 7 cm and a height of 14 cm. Use $\frac{22}{7}$ for π.

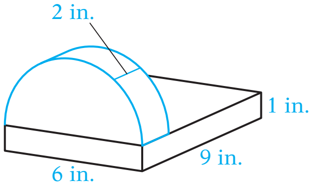
19. The length of a side of a cube is equal to the radius of a sphere. Which solid has the greater volume?

20. A sphere and a cylinder have the same radius. The height of the cylinder is equal to the radius of its base. Which solid has the greater volume?

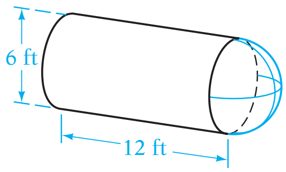
OBJECTIVE B *To find the volume of composite geometric solids*

For Exercises 21 to 26, find the volume. Use 3.14 for π.

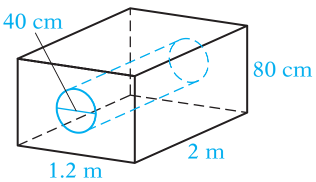
21.



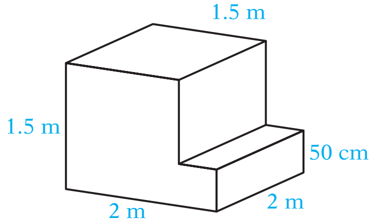
22.



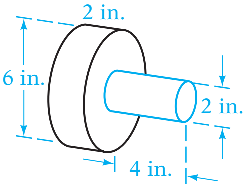
23.



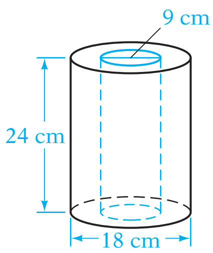
24.



25.



26.



For Exercises 27 and 28, use the solid shown in Exercise 26. If the solid is changed as described, will its volume increase or decrease?

27. The outer cylinder is changed to a rectangular solid with a square base. The height and width of the outer solid remain the same.

28. The inner cylinder is changed to a rectangular solid with a square base. The height and width of the inner solid remain the same.

OBJECTIVE C *To solve application problems*

For Exercises 29 to 42, solve. Use 3.14 for π.

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29. Fish Hatchery A rectangular tank at a fish hatchery is 9 m long, 3 m wide, and 1.5 m deep. Find the volume of the water in the tank when the tank is full.

30. Rocketry A fuel tank in a booster rocket is a cylinder 10 ft in diameter and 52 ft high. Find the volume of the fuel tank.

31. Ballooning A hot air balloon is in the shape of a sphere. Find the volume of a hot air balloon that is 32 ft in diameter. Round to the nearest hundredth.

32. Petroleum An oil tank, which is in the shape of a cylinder, is 4 m high and has a diameter of 6 m. The oil tank is two-thirds full. Find the number of cubic meters of oil in the tank. Round to the nearest hundredth.

33. Agriculture A silo, which is in the shape of a cylinder, is 16 ft in diameter and has a height of 30 ft. The silo is three-fourths full. Find the volume of the portion of the silo that is not being used for storage.

In the NEWS!

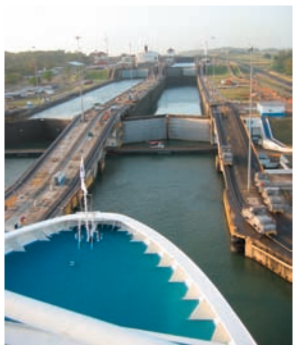
Super Bowl Win for Guacamole

Guacamole is the dish of choice at Super Bowl parties. If all the guacamole eaten during the Super Bowl were piled onto a football field—which, including endzones, is 360 ft long and 160 ft wide—it would cover the field to a depth of 3 ft!

Source: www.azcentral.com

34. Guacamole Consumption See the news clipping at the right. What is the volume, in cubic feet, of the guacamole eaten during the Super Bowl?

35. Guacamole Consumption See the news clipping at the right. Assuming that each person eats 1 c of guacamole, how many people eat guacamole during the Super Bowl? 1 $ft^3 \approx 119.68$ c.



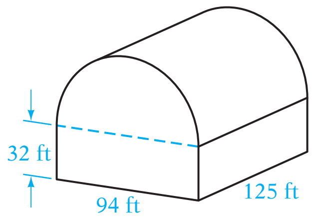
Jim Lipschutz/Shutterstock.com

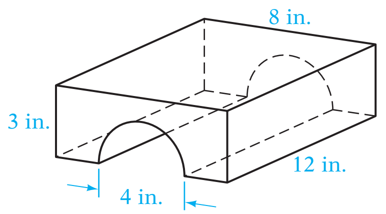
Panama Canal

36. The Panama Canal The Gatun Lock of the Panama Canal is 1000 ft long, 110 ft wide, and 60 ft deep. Find the volume of the lock in cubic feet.

37. The Panama Canal When the lock is full, the water in the Pedro Miguel Lock near the Pacific Ocean side of the Panama Canal fills a rectangular solid of dimensions 1000 ft long, 110 ft wide, and 43 ft deep. There are approximately 7.48 gal of water in each cubic foot. How many gallons of water are in the lock?

38. Architecture An architect is designing the heating system for an auditorium and needs to know the volume of the structure. Find the volume of the auditorium with the measurements shown in the figure.

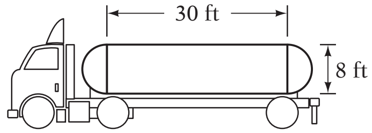




39. **Metal Works** Find the volume of the bushing shown at the right.

40. **Aquariums** How many gallons of water will fill a fish tank that is 12 in. long, 8 in. wide, and 9 in. high? Round to the nearest tenth. (1 gal = 231 in^3)

41. **Aquariums** How many gallons of water will fill an aquarium that is 12 in. wide, 18 in. long, and 16 in. high? Round to the nearest tenth. (1 gal = 231 in^3)

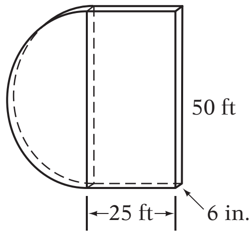


42. **Petroleum** A truck carrying an oil tank is shown in the figure at the right.

a. Without doing the calculations, determine whether the volume of the oil tank is more than 240 ft^3 or less than 240 ft^3.

b. If the tank is half full, how many cubic feet of oil is the truck carrying? Round to the nearest hundredth.

Construction For Exercises 43 to 46, use the diagram at the right showing the concrete floor of a building. State whether the given expression can be used to calculate the volume of the concrete floor in cubic feet.



43. $(25)(50)(6) + (0.5)(3.14)(25^2)(6)$

44. $(25)(50)(0.5) + (0.5)(3.14)(25^2)(0.5)$

45. $0.5[(25)(50) + (0.5)(3.14)(25^2)]$

46. $(25)(50)(0.5) + (0.5)(3.14)(50^2)(0.5)$

47. **Construction** Use the diagram for Exercises 43 to 46. At a cost of \$10 per cubic foot, find the cost of having the floor poured.

Critical Thinking

For Exercises 48 to 51, explain how you could cut through a cube so that the face of the resulting solid is the given geometric figure.

48. A square

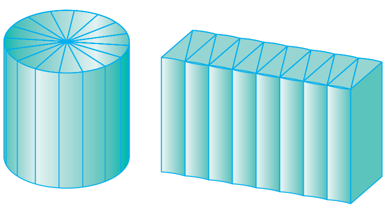
49. An equilateral triangle

50. A trapezoid

51. A hexagon

Projects or Group Activities

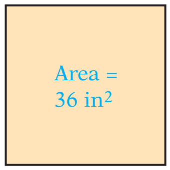
52. Suppose a cylinder is cut into 16 equal pieces, which are then arranged as shown at the right. The figure resembles a rectangular solid. What variable expressions could be used to represent the length, width, and height of the rectangular solid? Explain how the formula for the volume of a cylinder is derived from this approach.



SECTION

12.5 The Pythagorean Theorem

OBJECTIVE A *To find the square root of a number*



The area of a square is 36 in^2. What is the length of each side?

Area of the square = $(\text{side})^2$

$36 = \text{side} \cdot \text{side}$

What number multiplied times itself equals 36?

$36 = 6 \cdot 6$

Each side of the square is 6 in.

The **square root** of a number is one of two identical factors of the number. The square root symbol is $\sqrt{}$.

The square root of 36 is 6.

$\sqrt{36} = 6$

A **perfect square** is the product of a whole number times itself.

1, 4, 9, 16, 25, and 36 are perfect squares.

The square root of a perfect square is a whole number.

$1 \cdot 1 = 1$	$\sqrt{1} = 1$
$2 \cdot 2 = 4$	$\sqrt{4} = 2$
$3 \cdot 3 = 9$	$\sqrt{9} = 3$
$4 \cdot 4 = 16$	$\sqrt{16} = 4$
$5 \cdot 5 = 25$	$\sqrt{25} = 5$
$6 \cdot 6 = 36$	$\sqrt{36} = 6$

Point of Interest

The square root of a number that is not a perfect square is an irrational number. There is evidence of knowledge of irrational numbers dating from as early as 500 B.C. These numbers were not very well understood, and they were given the name *numerus surdus.* This phrase contains the Latin word *surdus,* which means “deaf” or “mute.” Thus irrational numbers were “inaudible numbers.”

If a number is not a perfect square, its square root can only be approximated. The approximate square roots of numbers can be found using a calculator. For example:

Number	*Square Root*
33	$\sqrt{33} \approx 5.745$
34	$\sqrt{34} \approx 5.831$
35	$\sqrt{35} \approx 5.916$

EXAMPLE 1

a. Find the square roots of the perfect squares 49 and 81.

b. Find the square roots of 27 and 108. Round to the nearest thousandth.

Solution

a. $\sqrt{49} = 7$ $\sqrt{81} = 9$

b. $\sqrt{27} \approx 5.196$ $\sqrt{108} \approx 10.392$

YOU TRY IT 1

a. Find the square roots of the perfect squares 16 and 169.

b. Find the square roots of 32 and 162. Round to the nearest thousandth.

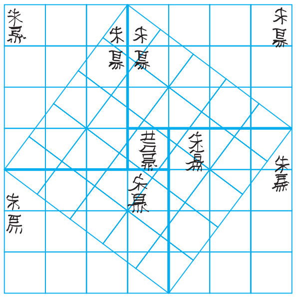
Your solution

Solution on p. S30

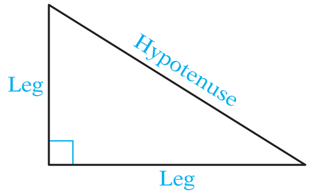
OBJECTIVE B *To find the unknown side of a right triangle using the Pythagorean Theorem*

Point of Interest

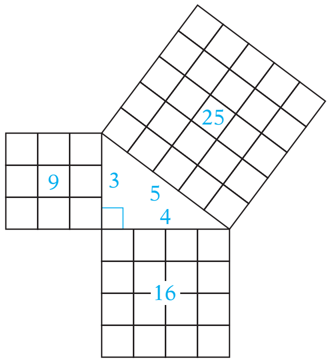
The first known proof of the Pythagorean Theorem is in a Chinese textbook that dates from 150 B.C. The book is called *Nine Chapters on the Mathematical Art.* The diagram below is from that book and was used in the proof of the theorem.



The Greek mathematician Pythagoras is generally credited with the discovery that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. This is called the **Pythagorean Theorem.** However, the Babylonians used this theorem more than 1000 years before Pythagoras lived.



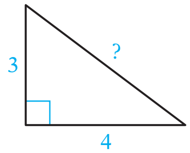
Square of the hypotenuse	equals	sum of the squares of the two legs
5^2	=	$3^2 + 4^2$
25	=	9 + 16
25	=	25



If the length of one side of a right triangle is unknown, then one of the following formulas can be used to find it.

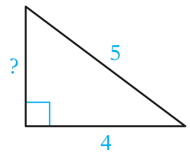
If the length of the hypotenuse is unknown, use

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{leg})^2}\\ &= \sqrt{(3)^2 + (4)^2}\\ &= \sqrt{9 + 16}\\ &= \sqrt{25} = 5\end{aligned}$$



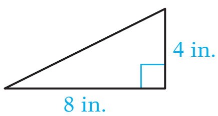
If the length of a leg is unknown, use

$$\begin{aligned}\text{Leg} &= \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}\\ &= \sqrt{(5)^2 - (4)^2}\\ &= \sqrt{25 - 16}\\ &= \sqrt{9} = 3\end{aligned}$$



EXAMPLE 2

Find the hypotenuse of the triangle shown in the figure. Round to the nearest thousandth.



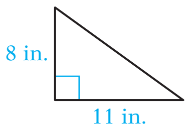
Solution

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{leg})^2}\\ &= \sqrt{8^2 + 4^2}\\ &= \sqrt{64 + 16}\\ &= \sqrt{80} \approx 8.944\end{aligned}$$

The hypotenuse is approximately 8.944 in.

YOU TRY IT 2

Find the hypotenuse of the triangle shown in the figure. Round to the nearest thousandth.

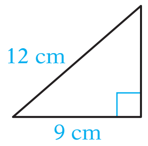


Your solution

Solution on p. S30

EXAMPLE 3

Find the length of the leg of the triangle shown in the figure. Round to the nearest thousandth.



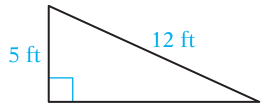
Solution

$$\text{Leg} = \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}$$
$$= \sqrt{12^2 - 9^2}$$
$$= \sqrt{144 - 81}$$
$$= \sqrt{63} \approx 7.937$$

The length of the leg is approximately 7.937 cm.

YOU TRY IT 3

Find the length of the leg of the triangle shown in the figure. Round to the nearest thousandth.



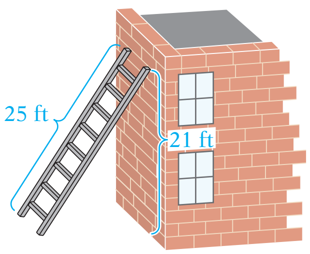
Your solution

Solution on p. S30

OBJECTIVE C ***To solve application problems***

EXAMPLE 4

A 25-foot ladder is placed against a building at a point 21 ft from the ground, as shown in the figure. Find the distance from the base of the building to the base of the ladder. Round to the nearest thousandth.



Strategy

To find the distance from the base of the building to the base of the ladder, use the Pythagorean Theorem. The hypotenuse is the length of the ladder (25 ft). One leg is the distance along the building from the ground to the top of the ladder (21 ft). The distance from the base of the building to the base of the ladder is the unknown leg.

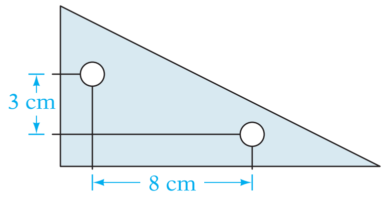
Solution

$$\text{Leg} = \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}$$
$$= \sqrt{25^2 - 21^2}$$
$$= \sqrt{625 - 441}$$
$$= \sqrt{184} \approx 13.565$$

The distance is approximately 13.565 ft.

YOU TRY IT 4

Find the distance between the centers of the holes in the metal plate shown in the figure. Round to the nearest thousandth.



Your strategy

Your solution

Solution on p. S30

12.5 EXERCISES

Concept Check

1. In the list below, identify those numbers that are perfect squares.

0, 1, 2, 8, 18, 49, 50, 64, 72, 81, 100

2. State the Pythagorean Theorem.

OBJECTIVE A *To find the square root of a number*

For Exercises 3 to 10, find the square root. Round to the nearest thousandth.

3. 7 **4.** 34 **5.** 42 **6.** 64

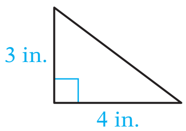
7. 165 **8.** 144 **9.** 189 **10.** 130

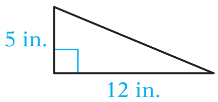
11. True or false? If a number is between 100 and 400, then its square root is between 10 and 20.

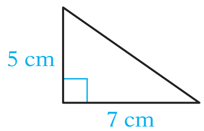
12. True or false? There are no perfect squares between 50 and 60.

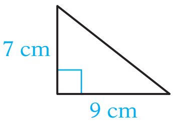
OBJECTIVE B *To find the unknown side of a right triangle using the Pythagorean Theorem*

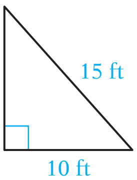
For Exercises 13 to 30, find the unknown side of the triangle. Round to the nearest thousandth.

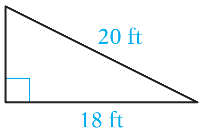
13.


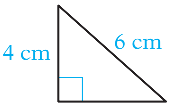
14.


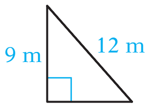
15.


16.


17.


18.


19.


20.


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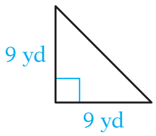
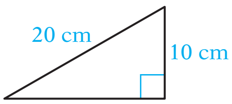
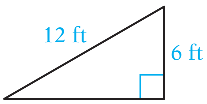
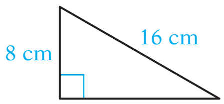
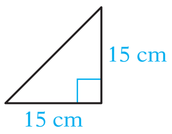


22.


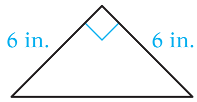
23.


24.


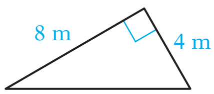
25.



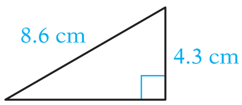
26.



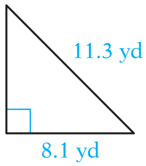
27.



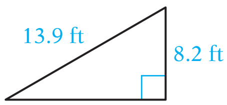
28.



29.



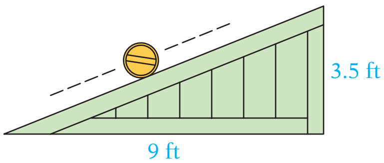
30.



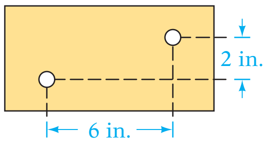
31. Describe a triangle for which the expression $\sqrt{50^2 - 40^2}$ could be used to find the length of one side of the triangle.

OBJECTIVE C *To solve application problems*

32. Ramps In the figure at the right, find the length of the ramp used to roll barrels up to the loading dock, which is 3.5 ft high. Round to the nearest hundredth.

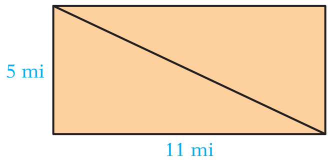


33. Metal Works Find the distance between the centers of the holes in the metal plate in the figure at the right. Round to the nearest hundredth.



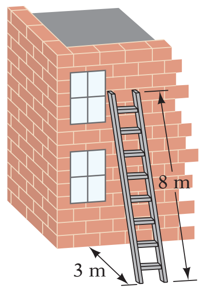
34. Travel If you travel 18 mi east and then 12 mi north, how far are you from your starting point? Round to the nearest tenth.

35. Travel If you travel 12 mi west and then 16 mi south, how far are you from your starting point?



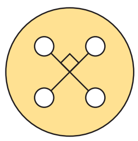
36. Geometry A diagonal of a rectangle is a line drawn from one vertex to the opposite vertex. Find the length of the diagonal in the rectangle shown at the right. Round to the nearest tenth.

37. Geometry A diagonal of a rectangle is a line drawn from one vertex to the opposite vertex. (See Exercise 36.) Find the length of a diagonal of a rectangle that has a length of 8 m and a width of 3.5 m. Round to the nearest tenth.

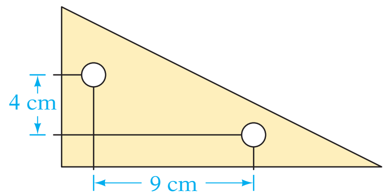


38. Home Maintenance A ladder 8 m long is placed against a building in preparation for washing the windows. How high on the building does the ladder reach when the bottom of the ladder is 3 m from the base of the building? Round to the nearest tenth.

39. Metal Works Four holes are drilled in the circular plate in the figure at the right. The centers of the holes are 3 in. from the center of the plate. Find the distance between the centers of adjacent holes. Round to the nearest thousandth.



40. **Metal Works** Find the distance between the centers of the holes in the metal plate shown in the diagram at the right. Round to the nearest tenth.

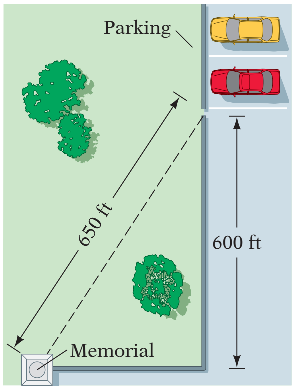


For Exercises 41 and 42, use the following information. A ladder c feet long leans against the side of a building with its bottom a feet from the building. The ladder reaches a height of b feet. Refer to the following lengths: **(i)** 15 ft **(ii)** 20 ft **(iii)** 30 ft

41. If $c = 18$ ft, which of the given lengths is a possible value for b?

42. If $b = 18$ ft, which of the given lengths are possible values for c?

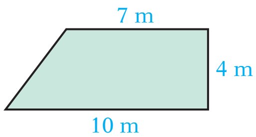
43. **Parks** An L-shaped sidewalk from a parking lot to a memorial is shown in the figure at the right. The distance directly across the grass to the memorial is 650 ft. The distance to the corner is 600 ft. Find the distance from the corner to the memorial.



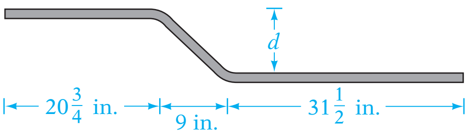
44. **Geometry** Find the perimeter of a right triangle with legs that measure 5 cm and 9 cm. Round to the nearest tenth.

45. **Geometry** Find the perimeter of a right triangle with legs that measure 6 in. and 10 in. Round to the nearest tenth.

46. **Landscaping** A vinyl fence is built around the plot shown in the figure at the right. At $12.90 per meter, how much did it cost to fence the plot? (*Hint:* Use the Pythagorean Theorem to find the unknown length.)

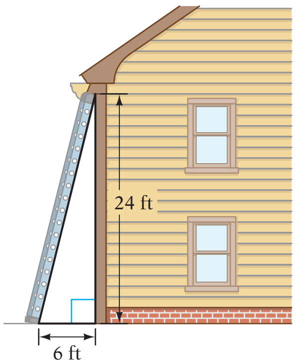


47. **Plumbing** Find the offset distance d of the length of pipe shown in the diagram below. The total length of the pipe is 62 in.

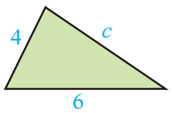


Critical Thinking

48. **Home Maintenance** You need to clean the gutters of your home. The gutters are 24 ft above the ground. For safety, the distance a ladder reaches up a wall should be four times the distance from the bottom of the ladder to the base of the house. Therefore, the ladder must be 6 ft from the base of the house. Will a 25-foot ladder be long enough to reach the gutters? Explain how you determined your answer.



49. Can the Pythagorean Theorem be used to find the length of side c of the triangle at the right? If so, determine c. If not, explain why the theorem cannot be used.



Projects or Group Activities

50. **a.** What is a Pythagorean triple? Provide three examples of Pythagorean triples.

b. There are formulas for creating Pythagorean triples. Find at least one of these formulas, and use it to generate several Pythagorean triples.

SECTION

12.6 Similar and Congruent Triangles

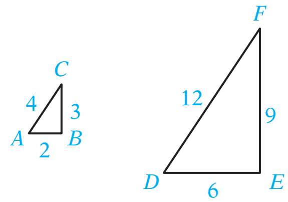
OBJECTIVE A *To solve similar and congruent triangles*

Morgan Lane Photography/ Shutterstock.com

Similar objects have the same shape but not necessarily the same size. A baseball is similar to a basketball. A model airplane is similar to an actual airplane.

Similar objects have corresponding parts; for example, the propellers on a model airplane correspond to the propellers on the actual airplane. The relationship between the sizes of the corresponding parts can be written as a ratio, and all such ratios will be the same. If the propellers on the model plane are $\frac{1}{50}$ the size of the propellers on the actual plane, then the model wing is $\frac{1}{50}$ the size of the actual wing, the model fuselage is $\frac{1}{50}$ the size of the actual fuselage, and so on.

The two triangles *ABC* and *DEF* shown at the right are similar. The ratios of corresponding sides are equal.



$$\frac{AB}{DE} = \frac{2}{6} = \frac{1}{3}, \frac{BC}{EF} = \frac{3}{9} = \frac{1}{3}, \text{ and } \frac{AC}{DF} = \frac{4}{12} = \frac{1}{3}$$

The ratio of corresponding sides is $\frac{1}{3}$.

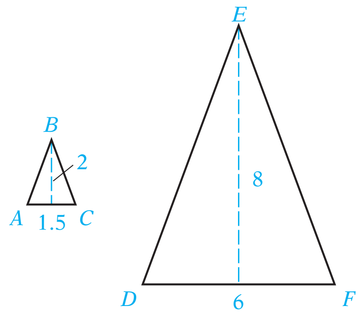
Because the ratios of corresponding sides are equal, three proportions can be formed:

$$\frac{AB}{DE} = \frac{BC}{EF}, \frac{AB}{DE} = \frac{AC}{DF}, \text{ and } \frac{BC}{EF} = \frac{AC}{DF}.$$

The ratio of the heights of two similar triangles is equal to the ratio of the corresponding sides, as shown in the figure below.

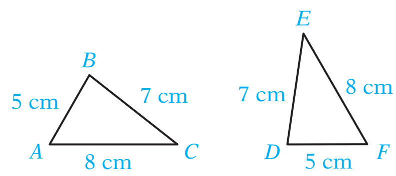
$$\text{Ratio of corresponding sides} = \frac{1.5}{6} = \frac{1}{4}$$

$$\text{Ratio of heights} = \frac{2}{8} = \frac{1}{4}$$



Congruent objects have the same shape *and* the same size.

The two triangles shown are congruent. They have exactly the same size.



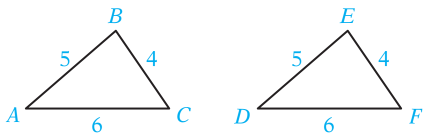
For triangles, *congruent* means that the corresponding sides *and* angles of the triangle are equal (this contrasts with similar triangles, in which corresponding angles, but not necessarily corresponding sides, are equal).

Here are two rules that can be used to determine whether two triangles are congruent.

Side-Side-Side (SSS) Rule

Two triangles are congruent if all three sides of one triangle equal the corresponding sides of the second triangle.

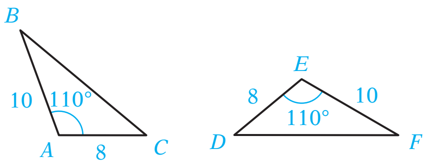
In the two triangles at the right, $AB = DE$, $AC = DF$, and $BC = EF$. The corresponding sides of triangles ABC and DEF are equal. The triangles are congruent by the SSS rule.



Side-Angle-Side (SAS) Rule

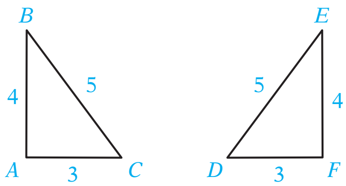
Two triangles are congruent if two sides and the included angle of one triangle equal the corresponding sides and included angle of the second triangle.

In the two triangles at the right, $AB = EF$, $AC = DE$, and $\angle CAB = \angle DEF$. The triangles are congruent by the SAS rule.



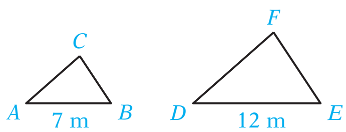
HOW TO 1 Determine whether the two triangles in the figure at the right are congruent.

Because $AC = DF$, $AB = FE$, and $BC = DE$, all three sides of one triangle equal the corresponding sides of the second triangle.
The triangles are congruent by the SSS rule.



EXAMPLE 1

Find the ratio of corresponding sides for the similar triangles ABC and DEF shown in the figure.

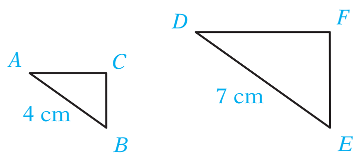


Solution

$$\frac{7\ \cancel{\text{m}}}{12\ \cancel{\text{m}}} = \frac{7}{12}$$

YOU TRY IT 1

Find the ratio of corresponding sides for the similar triangles ABC and DEF shown in the figure.

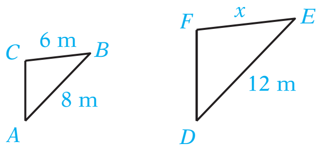


Your solution

Solution on p. S31

EXAMPLE 2

Triangles *ABC* and *DEF* in the figure are similar. Find *x*, the length of side *EF*.



Solution

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{8\ \not{m}}{12\ \not{m}} = \frac{6\text{ m}}{x}$$

$$8x = 12 \cdot 6\text{ m}$$

$$8x = 72\text{ m}$$

$$\frac{8x}{8} = \frac{72\text{ m}}{8}$$

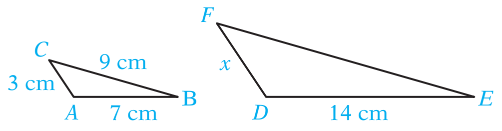
$$x = 9\text{ m}$$

• **The ratios of corresponding sides of similar triangles are equal.**

Side *EF* is 9 m.

YOU TRY IT 2

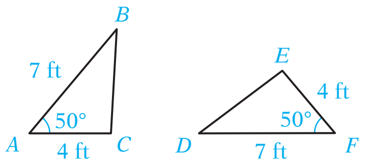
Triangles *ABC* and *DEF* in the figure are similar. Find *x*, the length of side *DF*.



Your solution

EXAMPLE 3

Determine whether triangle *ABC* in the figure is congruent to triangle *FDE*.

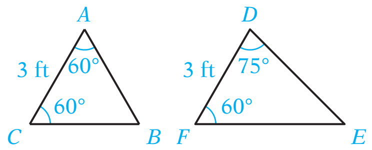


Solution

Because $AB = DF$, $AC = EF$, and angle $BAC =$ angle *DFE*, the triangles are congruent by the SAS rule.

YOU TRY IT 3

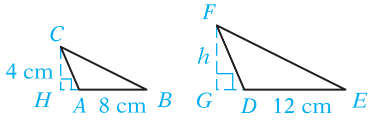
Determine whether triangle *ABC* in the figure is congruent to triangle *DEF*.



Your solution

EXAMPLE 4

Triangles *ABC* and *DEF* in the figure are similar. Find *h*, the height of triangle *DEF*.



Solution

$$\frac{8\ \not{cm}}{12\ \not{cm}} = \frac{4\text{ cm}}{h}$$

$$8h = 12 \cdot 4\text{ cm}$$

$$8h = 48\text{ cm}$$

$$\frac{8h}{8} = \frac{48\text{ cm}}{8}$$

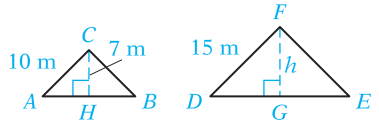
$$h = 6\text{ cm}$$

• **The ratios of corresponding sides of similar triangles equal the ratio of corresponding heights:** $\frac{AB}{DE} = \frac{CH}{FG}$.

The height of triangle *DEF* is 6 cm.

YOU TRY IT 4

Triangles *ABC* and *DEF* in the figure are similar. Find *h*, the height of triangle *DEF*.



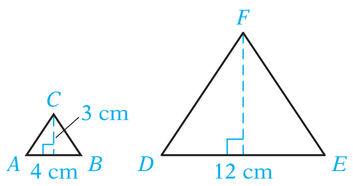
Your solution

Solutions on p. S31

OBJECTIVE B *To solve application problems*

EXAMPLE 5

Triangles *ABC* and *DEF* in the figure are similar. Find the area of triangle *DEF*.



Strategy

To find the area of triangle *DEF*:

- Solve a proportion to find the height of triangle *DEF*. Let $h =$ the height.
- Use the formula $A = \frac{1}{2}bh$.

Solution

$$\frac{AB}{DE} = \frac{\text{height of triangle } ABC}{\text{height of triangle } DEF}$$

$$\frac{4 \not{\text{cm}}}{12 \not{\text{cm}}} = \frac{3 \text{ cm}}{h}$$

$$4h = 12 \cdot 3 \text{ cm}$$

$$4h = 36 \text{ cm}$$

$$\frac{4h}{4} = \frac{36 \text{ cm}}{4}$$

$$h = 9 \text{ cm}$$

To height is 9 cm.

$$A = \frac{1}{2}bh$$

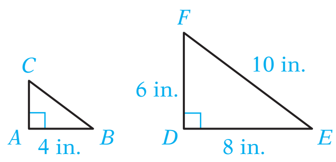
$$= \frac{1}{2}(12 \text{ cm})(9 \text{ cm})$$

$$= 54 \text{ cm}^2$$

The area is 54 cm^2.

YOU TRY IT 5

Triangles *ABC* and *DEF* in the figure are similar right triangles. Find the perimeter of triangle *ABC*.



Your strategy

Your solution

Solution on p. S31

12.6 EXERCISES

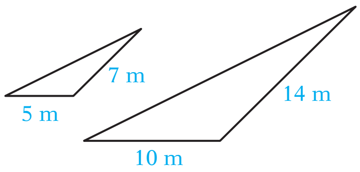
Concept Check

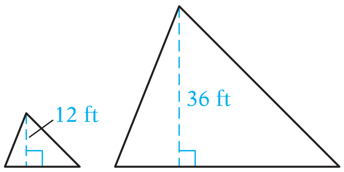
1. If two triangles are congruent, are they similar? If two triangles are similar, are they congruent?

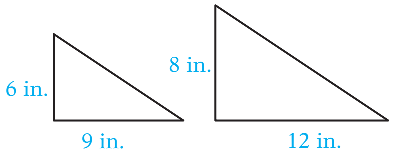
2. State whether the triangles must be congruent.
a. The corresponding angles of two triangles are equal.
b. The corresponding sides of two triangles are equal.

OBJECTIVE A *To solve similar and congruent triangles*

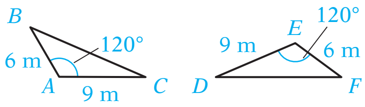
Find the ratio of corresponding sides for the similar triangles in Exercises 3 to 5.

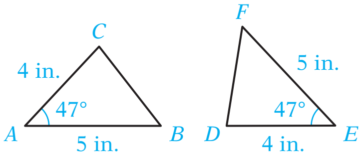
3.


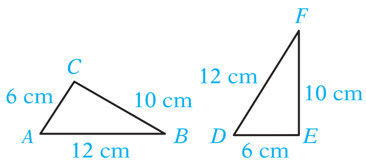
4.


5.


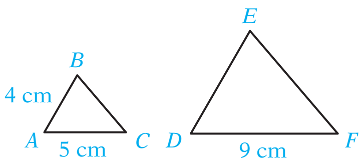
Determine whether the two triangles in Exercises 6 to 8 are congruent.

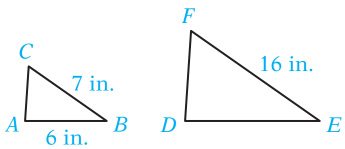
6.


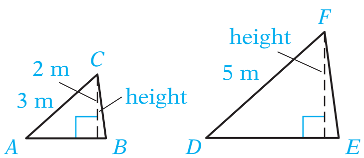
7.


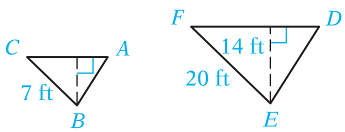
8.


Triangles *ABC* and *DEF* in Exercises 9 to 12 are similar. Find the indicated distance. Round to the nearest tenth.

9. Find side *DE*.


10. Find side *DE*.


11. Find the height of triangle *DEF*.


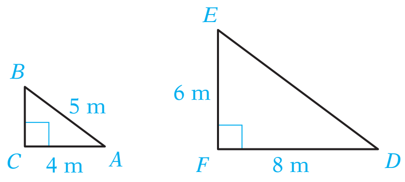
12. Find the height of triangle *ABC*.


13. True or false? If the ratio of the corresponding sides of two similar triangles is 1 to 1, then the two triangles are congruent.

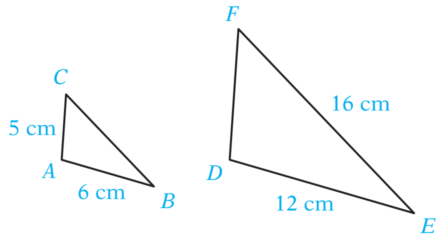
OBJECTIVE B *To solve application problems*

In Exercises 14 to 17, triangles *ABC* and *DEF* are similar.

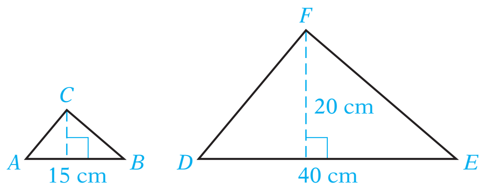
14. Find the perimeter of triangle *ABC*.



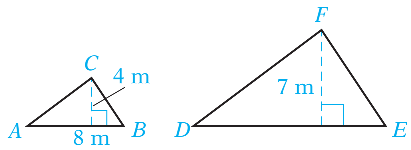
15. Find the perimeter of triangle *DEF*.



16. Find the area of triangle *ABC*.



17. Find the area of triangle *DEF*.



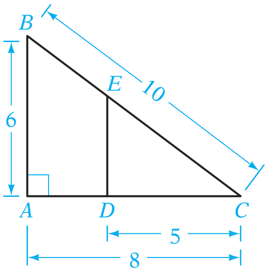
18. Determine whether the statement is always true, sometimes true, or never true.

a. If two angles of one triangle are equal to two angles of a second triangle, then the triangles are similar triangles.
b. Two isosceles triangles are similar triangles.
c. Two equilateral triangles are similar triangles.
d. The ratio of the perimeters of two similar triangles is the same as the ratio of corresponding sides of the two triangles.

Critical Thinking

19. Are all squares similar? Are all rectangles similar? Explain. Use a drawing in your explanation.

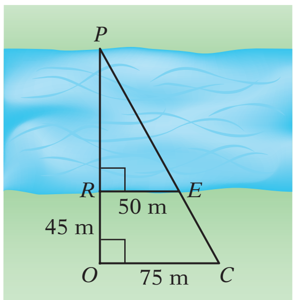
20. Figure *ABC* is a right triangle, and *DE* is parallel to *AB*. What is the perimeter of the trapezoid *ABED*?



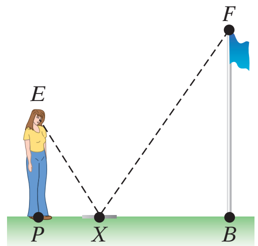
Projects or Group Activities

Some distances, such as the distance across a river or the distance to the top of a pole, can be measured indirectly by using similar triangles.

21. Find the distance *PR* across the river.



22. How can you determine the height of the pole by using a mirror placed at *X*? If $XB = 8$ ft, $PX = 4$ ft, and you are 5.5 ft tall, how tall is the pole?



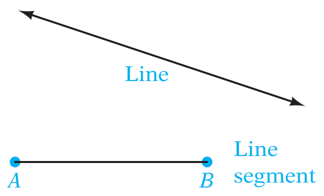
CHAPTER

12 Summary

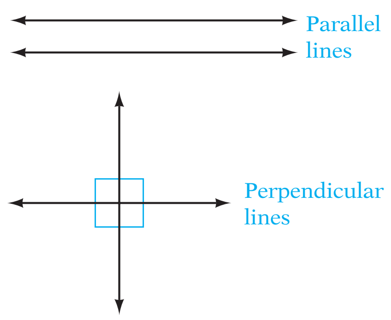
Key Words

Examples

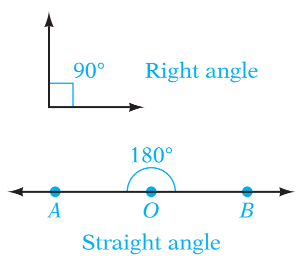
A **line** extends indefinitely in two directions. A **line segment** is part of a line and has two endpoints. The length of a line segment is the distance between the endpoints of the line segment. [12.1A, p. 522]



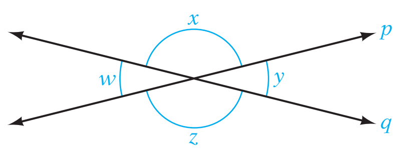
Parallel lines never meet; the distance between them is always the same. The symbol ∥ means "is parallel to." **Intersecting lines** cross at a point in the plane. **Perpendicular lines** are intersecting lines that form right angles. The symbol ⊥ means "is perpendicular to." [12.1A, pp. 522–523]



A **ray** starts at a point and extends indefinitely in one direction. An **angle** is formed when two rays start from the same point. The common point is called the **vertex** of the angle. An angle is measured in **degrees.** A 90° angle is a **right angle.** A 180° angle is a **straight angle. Complementary angles** are two angles whose sum is 90°. **Supplementary angles** are two angles whose sum is 180°. An **acute angle** is an angle whose measure is between 0° and 90°. An **obtuse angle** is an angle whose measure is between 90° and 180°. [12.1A, pp. 523–524]

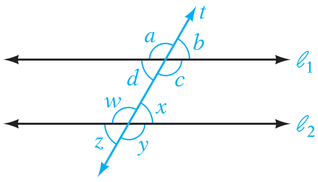


Two angles that are on opposite sides of the intersection of two lines are **vertical angles.** Vertical angles have the same measure. Two angles that share a common side are **adjacent angles.** Adjacent angles of intersecting lines are supplementary angles. [12.1C, p. 528]



Angles w and y are vertical angles.
Angles x and y are adjacent angles.

A line that intersects two other lines at two different points is a **transversal.** If the lines cut by a transversal are parallel lines, equal angles are formed: **alternate interior angles, alternate exterior angles,** and **corresponding angles.** [12.1C, pp. 528–529]



Parallel lines l_1 and l_2 are cut by transversal t. All four acute angles have the same measure. All four obtuse angles have the same measure.

A **quadrilateral** is a four-sided polygon. A **parallelogram,** a **rectangle,** and a **square** are quadrilaterals. [12.2A, p. 535]

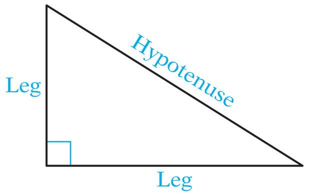
A **polygon** is a closed figure determined by three or more line segments. The line segments that form the polygon are its **sides.** A **regular polygon** is one in which each side has the same length and each angle has the same measure. Polygons are classified by the number of sides. [12.2A, p. 534]

Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

A **triangle** is a closed, three-sided plane figure. [12.1B, p. 525]

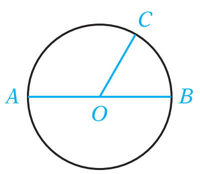
An **isosceles triangle** has two sides of equal length. The three sides of an **equilateral triangle** are of equal length. A **scalene triangle** has no two sides of equal length. An **acute triangle** has three acute angles. An **obtuse triangle** has one obtuse angle. [12.2A, p. 535]

A **right triangle** contains a right angle. The side opposite the right angle is called the **hypotenuse.** The other two sides are called **legs.** [12.1B, p. 525]



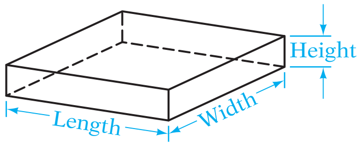
Right Triangle

A **circle** is a plane figure in which all points are the same distance from the center of the circle. The **diameter** of a circle is the length of a line segment through the center with endpoints on the circle. The **radius** of a circle is the length of a line segment from the center of the circle to a point on the circle. [12.1B, p. 526]



AB is a diameter of the circle.
OC is a radius.

Geometric solids are figures in space. Four common space figures are the rectangular solid, cube, sphere, and cylinder. A **rectangular solid** is a solid in which all six faces are rectangles. A **cube** is a rectangular solid in which all six faces are squares. A **sphere** is a solid in which all points on the surface are the same distance from the center of the sphere. The most common **cylinder** is one in which the bases are circles and are perpendicular to the side. [12.1B, pp. 526–527]

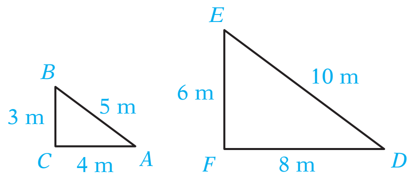


Rectangular Solid

The **square root** of a number is one of two identical factors of the number. The symbol for square root is $\sqrt{\ }$. A **perfect square** is the product of a whole number times itself. The square root of a perfect square is a whole number. [12.5A, p. 565]

$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, \ldots$
1, 4, 9, 16, . . . are perfect squares.
$\sqrt{1} = 1, \sqrt{4} = 2, \sqrt{9} = 3, \sqrt{16} = 4, \ldots$

Similar triangles have the same shape but not necessarily the same size. The ratios of corresponding sides are equal. The ratio of heights is equal to the ratio of corresponding sides. **Congruent triangles** have the same shape and the same size. [12.6A, p. 571]



Triangles *ABC* and *DEF* are similar. The ratio of corresponding side is $\frac{1}{2}$.

Essential Rules and Procedures

Examples

Angles of a Triangle [12.1B, p. 525]
The sum of the three angles of a triangle is 180°.

Two angles of a triangle measure 32° and 48°. Find the measure of the third angle.

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ \angle A + 32^\circ + 48^\circ &= 180^\circ \\ \angle A + 80^\circ &= 180^\circ \\ \angle A + 80^\circ - 80^\circ &= 180^\circ - 80^\circ \\ \angle A &= 100^\circ \end{aligned}$$

The measure of the third angle is 100°.

Formulas for Perimeter (the distance around a figure) [12.2A, pp. 536–537]

Triangle: $P = a + b + c$

Square: $P = 4s$

Rectangle: $P = 2L + 2W$

Circumference of a circle: $C = \pi d$ or $C = 2\pi r$

The length of a rectangle is 8 m. The width is 5.5 m. Find the perimeter.

$$\begin{aligned} P &= 2L + 2W \\ P &= 2(8 \text{ m}) + 2(5.5 \text{ m}) \\ &= 16 \text{ m} + 11 \text{ m} \\ &= 27 \text{ m} \end{aligned}$$

The perimeter is 27 m.

Formulas for Area (the amount of surface in a region) [12.3A, pp. 546–547]

Triangle: $A = \frac{1}{2}bh$

Square: $A = s^2$

Rectangle: $A = LW$

Circle: $A = \pi r^2$

Find the area of a circle with a radius of 4 cm. Use 3.14 for π.

$$A = \pi r^2$$
$$A \approx 3.14(4 \text{ cm})^2$$
$$= 50.24 \text{ cm}^2$$

The area is approximately 50.24 cm^2.

Formulas for Volume (the amount of space inside a figure in space) [12.4A, pp. 555–556]

Rectangular solid: $V = LWH$

Cube: $V = s^3$

Sphere: $V = \frac{4}{3}\pi r^3$

Cylinder: $V = \pi r^2 h$

Find the volume of a cube that measures 3 in. on a side.

$$V = s^3$$
$$V = (3 \text{ in.})^3$$
$$= 27 \text{ in.}^3$$

The volume is 27 in^3.

Pythagorean Theorem [12.5B, p. 566]

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

If the length of one side of a right triangle is unknown, one of the following formulas can be used to find it.

If the length of the hypotenuse is unknown, use

$$\text{Hypotenuse} = \sqrt{(\text{leg})^2 + (\text{leg})^2}$$

If the length of a leg is unknown, use

$$\text{Leg} = \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}$$

Two legs of a right triangle measure 6 ft and 8 ft. Find the length of the hypotenuse.

$$\text{Hypotenuse} = \sqrt{(\text{leg})^2 + (\text{leg})^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10$$

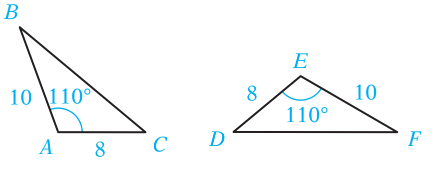
The length of the hypotenuse is 10 ft.

Side-Side-Side (SSS) Rule

Two triangles are congruent if the three sides of one triangle equal the corresponding sides of the second triangle.

Side-Angle-Side (SAS) Rule

Two triangles are congruent if two sides and the included angle of one triangle equal the corresponding sides and included angle of the second triangle. [12.6A, p. 572]



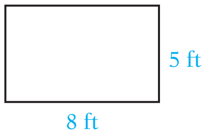
Triangles *ABC* and *EFD* are congruent by the SAS rule.

CHAPTER 12 Review Exercises

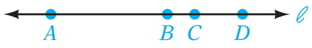
1. The diameter of a sphere is 1.5 m. Find the radius of the sphere.

2. Find the circumference of a circle with a radius of 5 cm. Use 3.14 for π.

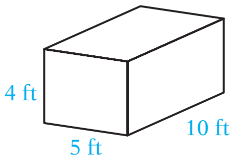
3. Find the perimeter of the rectangle in the figure below.



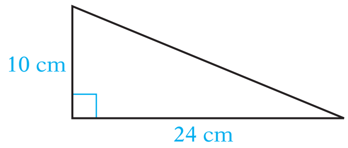
4. Given $AB = 15$, $CD = 6$, and $AD = 24$, find BC.



5. Find the volume of the rectangular solid shown below.



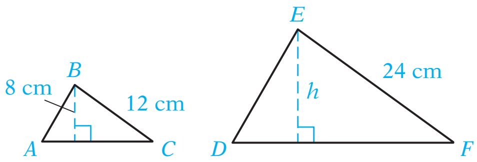
6. Find the unknown side of the triangle in the figure below.



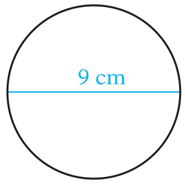
7. Find the supplement of a 105° angle.

8. Find the square root of 15. Round to the nearest thousandth.

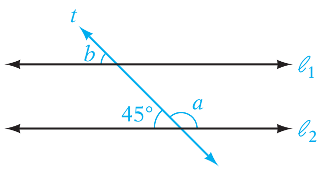
9. Triangles *ABC* and *DEF* are similar. Find the height of triangle *DEF*.



10. Find the area of the circle shown below. Use 3.14 for π.

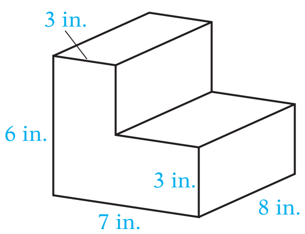


11. In the figure below, $\ell_1 \parallel \ell_2$.
a. Find the measure of angle *b*.
b. Find the measure of angle *a*.

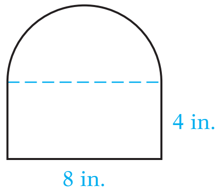


12. Find the area of the rectangle shown below.

13. Find the volume of the composite figure shown below.

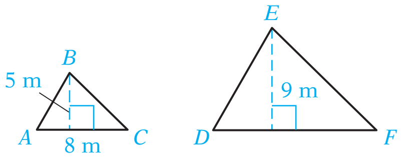


14. Find the area of the composite figure shown below. Use 3.14 for π.

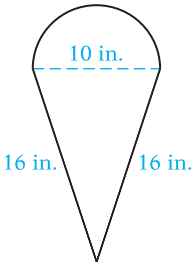


15. Find the volume of a sphere with a diameter of 8 ft. Use 3.14 for π. Round to the nearest tenth.

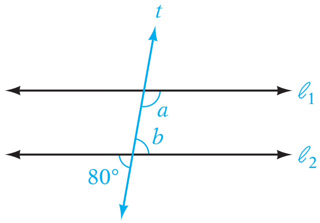
16. Triangles ABC and DEF are similar. Find the area of triangle DEF.



17. Find the perimeter of the composite figure shown below. Use 3.14 for π.



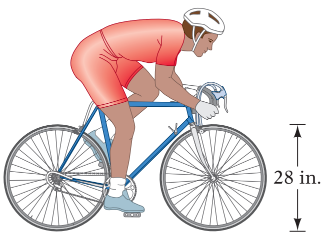
18. In the figure below, $\ell_1 \parallel \ell_2$.
a. Find the measure of angle b.
b. Find the measure of angle a.



19. **Home Maintenance** How high on a building will a 17-foot ladder reach when the bottom of the ladder is 8 ft from the building?

20. A right triangle has a 32° angle. Find the measures of the other two angles.

21. **Travel** A bicycle tire has a diameter of 28 in. How many feet does the bicycle travel when the wheel makes 10 revolutions? Use 3.14 for π. Round to the nearest tenth of a foot.



22. **Aquariums** Use the information in the news clipping at the right.
a. Inside and out, what is the total area of window that must be cleaned?
b. What volume, in cubic inches, does the window in the exhibit fill?

Charles Crust/Danita Delimont.com "Danita Delimont Photography"/Newscom

In the NEWS!

Dive Shows at Aquarium

Visitors to the Window on Washington Waters exhibit at the Seattle Aquarium can choose from three daily dive shows. Divers in the 120,000-gallon tank talk to visitors through a 40-foot by 20-foot rectangular acrylic window that is 12.5 in. thick.

Source: Seattle Aquarium

23. **Agriculture** A silo in the shape of a cylinder is 9 ft in diameter and has a height of 18 ft. Find the volume of the silo. Use 3.14 for π.

24. Find the area of a right triangle with a base of 8 m and a height of 2.75 m.

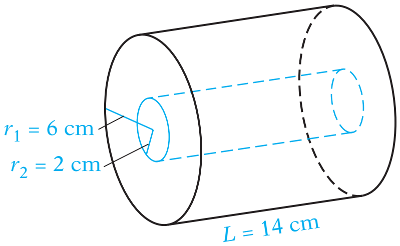
25. **Travel** If you travel 20 mi west and then 21 mi south, how far are you from your starting point?

CHAPTER 12 TEST

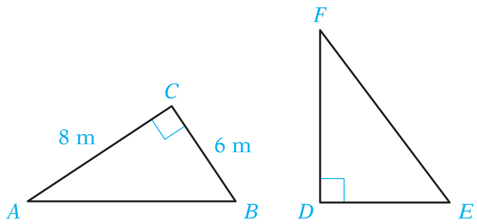
1. Find the volume of a cylinder with a height of 6 m and a radius of 3 m. Use 3.14 for π.

2. Find the perimeter of a rectangle that has a length of 2 m and a width of 1.4 m.

3. Find the volume of the composite figure. Use 3.14 for π.



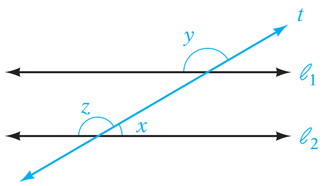
4. Triangles *ABC* and *FED* are congruent right triangles. Find the length of *FE*.



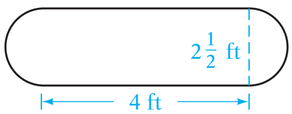
5. Find the complement of a 32° angle.

6. Find the area of a circle that has a diameter of 2 m. Use $\frac{22}{7}$ for π.

7. In the figure below, lines ℓ_1 and ℓ_2 are parallel. Angle *x* measures 30°. Find the measure of angle *y*.

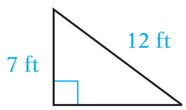


8. Find the perimeter of the composite figure. Use 3.14 for π.

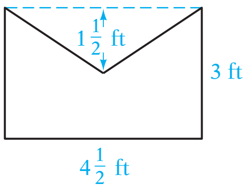


9. Find the square root of 189. Round to the nearest thousandth.

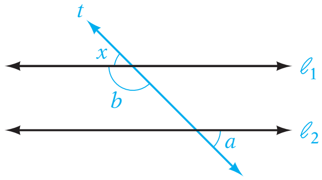
10. Find the unknown side of the triangle shown below. Round to the nearest thousandth.



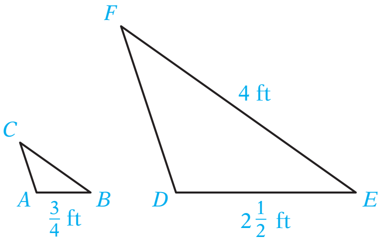
11. Find the area of the composite figure.



12. In the figure below, lines ℓ_1 and ℓ_2 are parallel. Angle x measures 45°. Find the measures of angles a and b.

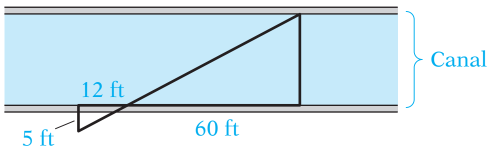


13. Triangles ABC and DEF are similar. Find side BC.



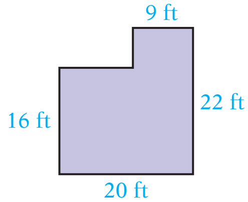
14. A right triangle has a 40° angle. Find the measures of the other two angles.

15. Measurement Use similar triangles to find the width of the canal shown in the figure at the right.

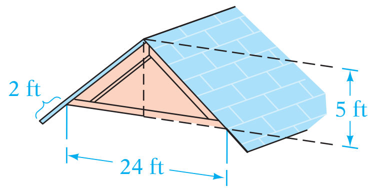


16. Consumerism How much more pizza is contained in a pizza with radius 10 in. than in one with radius 8 in.? Use 3.14 for π.

17. Interior Design A carpet is to be placed as shown in the diagram at the right. At \$26.80 per square yard, how much will it cost to carpet the area? Round to the nearest cent. ($9\text{ ft}^2 = 1\text{ yd}^2$)



18. Construction Find the length of the rafter needed for the roof shown in the figure.



19. Forestry Find the cross-sectional area of a redwood tree that is 11 ft 6 in. in diameter. Use 3.14 for π. Round to the nearest hundredth.

20. Alcatraz Inmate cells at Alcatraz were 9 ft long and 5 ft wide. The height of a cell was 7 ft.

a. Find the area of the floor of a cell at Alcatraz.

b. Find the volume of a cell at Alcatraz.

iStockphoto.com/slobo

Cumulative Review Exercises

1. Find the GCF of 96 and 144.

2. Add: $3\frac{5}{12} + 2\frac{9}{16} + 1\frac{7}{8}$

3. Find the quotient of $4\frac{1}{3}$ and $6\frac{2}{9}$.

4. Simplify: $\left(\frac{2}{3}\right)^2 \div \left(\frac{1}{3} + \frac{1}{2}\right) - \frac{2}{5}$

5. Simplify: $-\frac{2}{3} - \left(-\frac{5}{8}\right)$

6. Write "\$348.80 earned in 20 hours" as a unit rate.

7. Solve the proportion $\frac{3}{8} = \frac{n}{100}$.

8. Write $37\frac{1}{2}\%$ as a fraction.

9. Evaluate $a^2 - (b^2 - c)$ when $a = 2$, $b = -2$, and $c = -4$.

10. 30.94 is 36.4% of what number?

11. Solve: $\frac{x}{3} + 3 = 1$

12. Solve: $2(x - 3) + 2 = 5x - 8$

13. Convert 32.5 km to meters.

14. Subtract: 32 m − 42 cm

15. Solve: $\frac{2}{3}x = -10$

16. Solve: $2x - 4(x - 3) = 8$

17. **Finance** You bought a car for \$26,488 and made a down payment of \$1000. You paid the balance in 36 equal monthly installments. Find the monthly payment.

18. **Taxes** The sales tax on a color printer costing \$175 is \$6.75. At the same rate, find the sales tax on a home theater system costing \$1220.

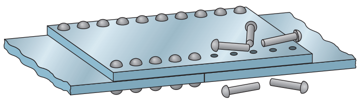
19. Compensation A heavy-equipment operator receives an hourly wage of \$32.12 after receiving a 10% wage increase. Find the operator's hourly wage before the increase.

20. Discount A hardware store is advertising a discount rate of 55% on windows. Find the sale price of a window that has a regular price of \$240.

21. Investments An IRA pays 7% annual interest, compounded daily. What will be the value of an investment of \$25,000 after 20 years? Use the table in the Appendix.

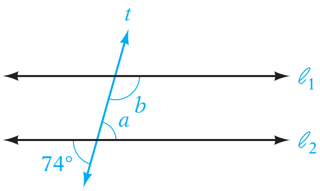
22. Shipping A square tile measuring 4 in. by 4 in. weighs 6 oz. Find the weight, in pounds, of a package of 144 such tiles.

23. Metal Works Twenty rivets, in two rows, are used to fasten two steel plates together. The plates are 5.4 m long, and the rivets are equally spaced with a rivet at each end. Find the distance, in centimeters, between the rivets in one row.

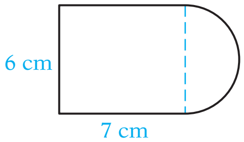


24. Integer Problems The total of four times a number and two is negative six. Find the number.

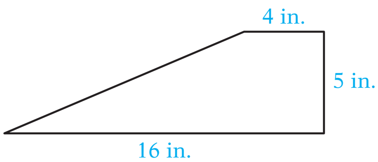
25. The lines ℓ_1 and ℓ_2 in the figure below are parallel.
a. Find the measure of angle a.
b. Find the measure of angle b.



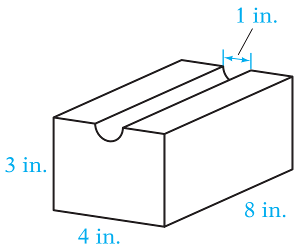
26. Find the perimeter of the composite figure. Use 3.14 for π.



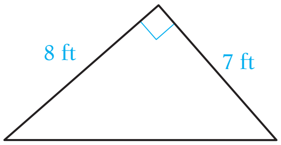
27. Find the area of the composite figure.



28. Find the volume of the composite figure. Use 3.14 for π.



29. Find the unknown side of the triangle shown in the figure below. Round to the nearest hundredth.



30. Triangles ABC and FED below are similar. Find the perimeter of FED.

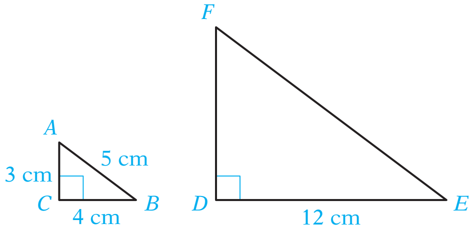


Table of Measurements

Prefixes in the Metric System of Measurement

kilo-	1000	deci-	0.1
hecto-	100	centi-	0.01
deca-	10	milli-	0.001

Metric System

Length

mm	millimeters
cm	centimeters
m	meters
km	kilometers

Capacity

ml	milliliters
cl	centiliters
L	liters
kl	kiloliters

Weight/Mass

mg	milligrams
cg	centigrams
g	grams
kg	kilograms

Area

cm^2	square centimeters
m^2	square meters
km^2	square kilometers

Rate

m/s	meters per second
km/s	kilometers per second
km/h	kilometers per hour

Time

h	hours
min	minutes
s	seconds

Measurement Abbreviations
U.S. Customary System

Length

in.	inches
ft	feet
yd	yards
mi	miles

Capacity

oz	fluid ounces
c	cups
qt	quarts
gal	gallons

Weight

oz	ounces
lb	pounds

Area

in^2	square inches
ft^2	square feet
yd^2	square yards
mi^2	square miles

Rate

ft/s	feet per second
mi/h	miles per hour

Time

h	hours
min	minutes
s	seconds

SECTION 11.5

1. No **3.** $y - 9$ **5.** $z + 3$ **7.** $\frac{2}{3}n + n$ **9.** $\frac{m}{m - 3}$ **11.** $9(x + 4)$ **13.** $x - \frac{x}{2}$ **15.** $\frac{z - 3}{z}$ **17.** $2(t + 6)$ **19.** $\frac{x}{9 + x}$ **21.** $3(b + 6)$ **23. a.** 3 more than twice x **b.** Twice the sum of x and 3 **25.** x^2 **27.** $\frac{x}{20}$ **29.** $4x$ **31.** $\frac{3}{4}x$ **33.** $4 + x$ **35.** $5x - x$ **37.** $x(x + 2)$ **39.** $7(x + 8)$ **41.** $x^2 + 3x$ **43.** $(x + 3) + \frac{1}{2}x$ **45.** No **47.** $\frac{a + 3}{4}$ **49.** $\frac{4c}{7} - 9$ **51.** $2x$

SECTION 11.6

1. No **3.** $x + 7 = 12; 5$ **5.** $3x = 18; 6$ **7.** $x + 5 = 3; -2$ **9.** $6x = 14; \frac{7}{3}$ **11.** $\frac{5}{6}x = 15; 18$ **13.** $3x + 4 = 8; \frac{4}{3}$ **15.** $\frac{1}{4}x - 7 = 9; 64$ **17.** $\frac{x}{9} = 14; 126$ **19.** $\frac{x}{4} - 6 = -2; 16$ **21.** $7 - 2x = 13; -3$ **23.** $9 - \frac{x}{2} = 5; 8$ **25.** $\frac{3}{5}x + 8 = 2; -10$ **27.** $\frac{x}{4.18} - 7.92 = 12.52; 85.4392$ **29.** No **31.** n represents the median price of a house in 2005. **33.** The length of the Brooklyn Bridge is 486 m. **35.** The Army paid $444 million in re-enlistment bonuses in 2010. **37.** The value of the SUV last year was $20,000. **39.** Infants aged 3 months to 11 months sleep 12.7 h each day. **41.** Five years ago the calculator cost $96. **43.** The recommended daily allowance of sodium is 2.5 g. **45.** About 9230 species of animals were known to be at risk of extinction. **47.** Americans consume 20 billion hot dogs annually. **49.** It took 3 h to install the water softener. **51.** On average, U.S. workers take 13 vacation days per year. **53.** The total sales for the month were $42,540. **55.** $\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$ **57.** $s = 16t^2$

CHAPTER 11 REVIEW EXERCISES

1. $-2a + 2b$ [11.1C] **2.** Yes [11.2A] **3.** -4 [11.2B] **4.** 7 [11.3A] **5.** 13 [11.1A] **6.** -9 [11.2C] **7.** -18 [11.3A] **8.** $-3x + 4$ [11.1C] **9.** 5 [11.4A] **10.** -5 [11.2B] **11.** No [11.2A] **12.** 6 [11.1A] **13.** 5 [11.4B] **14.** 10 [11.3A] **15.** $-4bc$ [11.1B] **16.** $-\frac{5}{2}$ [11.4A] **17.** $\frac{5}{4}$ [11.2C] **18.** $\frac{71}{30}x^2$ [11.1B] **19.** $-\frac{1}{3}$ [11.4B] **20.** $\frac{54}{5}$ [11.3A] **21.** The car averaged 23 mi/gal. [11.2D] **22.** The temperature is 37.8°C. [11.3B] **23.** $n + \frac{n}{5}$ [11.5A] **24.** $(n + 5) + \frac{1}{3}n$ [11.5B] **25.** The number is 2. [11.6A] **26.** The number is 10. [11.6A] **27.** The regular price of the tablet PC is $490. [11.6B] **28.** Last year's crop was 25,300 bushels. [11.6B]

CHAPTER 11 TEST

1. 95 [11.3A; HOW TO 1] **2.** 26 [11.2B; HOW TO 2] **3.** $-11x - 4y$ [11.1B; HOW TO 3] **4.** $\frac{16}{5}$ [11.4A; HOW TO 1] **5.** -2 [11.3A; Example 1] **6.** -38 [11.1A; HOW TO 1] **7.** No [11.2A; Example 2] **8.** $-14ab + 9$ [11.1B; HOW TO 2] **9.** $-\frac{14}{5}$ [11.2C; Example 6] **10.** $8y - 7$ [11.1C; Example 9] **11.** 0 [11.4B; Example 2] **12.** $-\frac{1}{2}$ [11.3A; Example 2] **13.** $-\frac{35}{6}$ [11.1A; Example 4] **14.** -16 [11.2C; Example 7] **15.** -3 [11.3A; Example 2] **16.** 11 [11.4B; Example 3] **17.** The monthly payment is $137.50. [11.2D; You Try It 10] **18.** 4000 clocks were made during the month. [11.3B; Example 4] **19.** The time required is 11.5 s. [11.3B; Example 3] **20.** $x + \frac{1}{3}x$ [11.5A; Example 1] **21.** $5(x + 3)$ [11.5B; HOW TO 2] **22.** $2x - 3 = 7; 5$ [11.6A; HOW TO 1] **23.** The number is $-\frac{7}{2}$. [11.6A; HOW TO 1] **24.** Eduardo's total sales for the month were $40,000. [11.6B; You Try It 8] **25.** The mechanic worked for 3 h. [11.6B; Example 7]

CUMULATIVE REVIEW EXERCISES

1. 41 [1.6B] **2.** $1\frac{7}{10}$ [2.5C] **3.** $\frac{35}{72}$ [2.8B] **4.** 0.047383 [3.4A] **5.** $9.10/h [4.2B] **6.** 26.67 [4.3B] **7.** $\frac{4}{75}$ [5.1A] **8.** 140% [5.3A] **9.** 6.4 [5.4A] **10.** 18 ft 9 in. [8.1B] **11.** 22 oz [8.2A] **12.** 0.282 g [9.2A] **13.** -1 [10.2A] **14.** 19 [10.2B] **15.** 6 [10.5B] **16.** -48 [11.1A] **17.** $-10x + 8z$ [11.1B] **18.** $3y + 23$ [11.1C] **19.** -1 [11.3A] **20.** $-\frac{25}{4}$ [11.4B] **21.** $-\frac{15}{2}$ [11.2C] **22.** -21 [11.3A] **23.** The percent is 17.6%. [5.3B] **24.** The price of the piece of pottery is $39.90. [6.2B] **25. a.** The discount is $81. **b.** The discount rate is 18%. [6.2D] **26.** The simple interest due on the loan is $2933.33. [6.3A] **27.** 2277 people participated in the survey. [5.2B] **28.** The probability is $\frac{1}{8}$ that the sum of the dots on the upward faces on the two dice is 7. [7.5A] **29.** The total sales were $32,500. [11.3B] **30.** The number is 2. [11.6A]

Answers to Chapter 12 Selected Exercises

PREP TEST

1. 43 [11.2B] **2.** 51 [11.2B] **3.** 56 [1.6B] **4.** 56.52 [11.1A] **5.** 113.04 [11.1A] **6.** 14.4 [4.3B]

SECTION 12.1

1. 0°; 90° **3.** 180° **5.** perpendicular **7.** Hypotenuse **9.** 30 **11.** 21 **13.** 14 **15.** 59° **17.** 108° **19.** 77° **21.** 53° **23.** An acute angle **25.** An obtuse angle **27.** 77° **29.** 118° **31.** 133° **33.** 86° **35.** Square **37.** Circle **39.** 102° **41.** 90° and 45° **43.** 14° **45.** 90° and 65° **47.** 8 in. **49.** $4\frac{2}{3}$ ft **51.** 7 cm **53.** 2 ft 4 in. **55.** False **57.** True **59.** $\angle a = 131°, \angle b = 49°$ **61.** $\angle a = 131°, \angle b = 49°$ **63.** $\angle a = 44°, \angle b = 44°$ **65.** $\angle a = 55°, \angle b = 125°$ **67.** $\angle a = 105°, \angle b = 75°$ **69.** $\angle a = 62°, \angle b = 118°$ **71.** True

SECTION 12.2

1. a. triangle **b.** hexagon **c.** rhombus; square **d.** two **3.** $P = 2L + 2W$ **5.** 56 in. **7.** 20 ft **9.** 47.1 cm **11.** 92 cm **13.** 9 ft 10 in. **15.** 50.24 cm **17.** 240 m **19.** The trainer will need 270 ft of fencing. **21.** Perimeter of the square **23.** 121 cm **25.** 50.56 m **27.** 3.57 ft **29.** 139.3 m **31.** Less than **33.** The amount of fencing needed is 60 ft. **35.** The amount of binding needed is 24 ft. **37.** The circumferene of the track was 990 ft. **39.** Two packages of bias binding are needed. **41.** The tricycle travels 25.12 ft. **43.** The perimeter of the roller rink is 81.4 m. **45.** The length of the weather stripping is 20.71 ft. **47. a.** The circumference is two times larger. **b.** The circumference is two times larger.

SECTION 12.3

1. a. $A = LW$ **b.** $A = \pi r^2$ **3.** 144 ft^2 **5.** 81 in^2 **7.** 50.24 ft^2 **9.** 20 in^2 **11.** 2.13 cm^2 **13.** 16 ft^2 **15.** 817 in^2 **17.** 154 in^2 **19.** The area is 8100 ft^2. **21.** Equal to **23.** 26 cm^2 **25.** 2220 cm^2 **27.** 150.72 in^2 **29.** 8.851323 ft^2 **31.** Equal to **33.** 7500 yd^2 of artificial turf must be purchased. **35.** You should buy 2 qt of stain. **37.** The area watered by the irrigation system is approximately 7850 ft^2. **39.** No, the expression cannot be used to calculate the area of the carpet. **41.** Yes, the expression can be used to calculate the area of the carpet. **43.** It will cost $986.10 to carpet the area. **45.** You should purchase 48 tiles. **47.** The area is 68 m^2. **49. a.** The area of the rink is more than 8000 ft^2. **b.** 19,024 ft^2 of hardwood flooring is needed to cover the rink. **51.** The cost is $1920. **55.** The area of Lake Tahoe is approximately 222.2 mi^2.

CHECK YOUR PROGRESS: CHAPTER 12

1. 28 [12.1A] **2.** 63°; 153° [12.1A] **3.** 68° [12.1A] **4.** $\angle a = 120°$, $\angle b = 60°$ [12.1C] **5.** 68° and 90° [12.1B] **6.** 14° [12.1B] **7.** 53° [12.1A] **8.** $\angle a = 130°$, $\angle b = 50°$ [12.1C] **9.** 8 m [12.2A] **10.** 22.608 in. [12.2A] **11.** 6.056 m [12.2B] **12.** 6 m^2 [12.3A] **13.** 28.26 in^2 [12.3A] **14.** 41.12 cm^2 [12.3B] **15.** The cost of the carpet is $546. [12.2C] **16.** The area of the walkway is 392.5 ft^2. [12.3C]

SECTION 12.4

1. a. $V = s^3$ **b.** $V = \frac{4}{3}\pi r^3$ **c.** $V = \pi r^2 h$ **3.** 144 cm^3 **5.** 512 in^3 **7.** 2143.57 in^3 **9.** 150.72 cm^3 **11.** 6.4 m^3 **13.** 5572.45 mm^3 **15.** 3391.2 ft^3 **17.** $42\frac{7}{8}$ ft^3 **19.** Sphere **21.** 82.26 in^3 **23.** 1.6688 m^3 **25.** 69.08 in^3 **27.** Increase **29.** The volume of the water in the tank is 40.5 m^3. **31.** The volume of the hot air balloon is approximately 17,148.59 ft^3. **33.** The volume of the portion of the silo not being used for storage is 1507.2 ft^3. **35.** 20,680,704 people eat guacamole during the Super Bowl. **37.** The lock contains 35,380,400 gal of water. **39.** The volume of the bushing is approximately 212.64 in^3. **41.** The tank will hold 15.0 gal. **43.** No, the expression cannot be used to calculate the volume of the concrete floor. **45.** Yes, the expression can be used to calculate the volume of the concrete floor. **47.** The cost of having the floor poured is $11,156.25.

SECTION 12.5

1. 0, 1, 49, 64, 81, 100 **3.** 2.646 **5.** 6.481 **7.** 12.845 **9.** 13.748 **11.** True **13.** 5 in. **15.** 8.602 cm **17.** 11.180 ft **19.** 4.472 cm **21.** 12.728 yd **23.** 10.392 ft **25.** 21.213 cm **27.** 8.944 m **29.** 7.879 yd **31.** A right triangle with hypotenuse of length 50 units and a leg of length 40 units **33.** The distance is 6.32 in. **35.** You are 20 mi from your starting point. **37.** The length of a diagonal is 8.7 m. **39.** The distance is 4.243 in. **41.** i **43.** The distance is 250 ft. **45.** The perimeter is 27.7 in. **47.** The offset distance is $3\frac{3}{4}$ in.

SECTION 12.6

1. Yes; no **3.** $\frac{1}{2}$ **5.** $\frac{3}{4}$ **7.** Congruent **9.** 7.2 cm **11.** 3.3 m **13.** True **15.** The perimeter of triangle *DEF* is 38 cm. **17.** The area of triangle *DEF* is 49 m^2. **21.** The distance is 90 m.

CHAPTER 12 REVIEW EXERCISES

1. 0.75 m [12.1B] **2.** 31.4 cm [12.2A] **3.** 26 ft [12.2A] **4.** 3 [12.1A] **5.** 200 ft^3 [12.4A] **6.** 26 cm [12.5B] **7.** 75° [12.1A] **8.** 3.873 [12.5A] **9.** 16 cm [12.6A] **10.** 63.585 cm^2 [12.3A] **11. a.** 45° **b.** 135° [12.1C] **12.** 55 m^2 [12.3A] **13.** 240 in^3 [12.4B] **14.** 57.12 in^2 [12.3B] **15.** 267.9 ft^3 [12.4A] **16.** 64.8 m^2 [12.6B] **17.** 47.7 in. [12.2B] **18. a.** 80° **b.** 100° [12.1C] **19.** The ladder will reach 15 ft up the building. [12.5C] **20.** 90° and 58° [12.1B] **21.** The bicycle travels approximately 73.3 ft in 10 revolutions. [12.2C] **22. a.** The total area of window glass that must be cleaned is 1600 ft^2. [12.3C] **b.** The window in the exhibit fills 1,440,000 in^3. [12.4C] **23.** The volume of the silo is approximately 1144.53 ft^3. [12.4C] **24.** The area is 11 m^2. [12.3A] **25.** The distance from the starting point is 29 mi. [12.5C]

CHAPTER 12 TEST

1. 169.56 m^3 [12.4A; Example 3] **2.** 6.8 m [12.2A; Example 1] **3.** 1406.72 cm^3 [12.4B; Example 6] **4.** 10 m [12.6A; You Try It 5] **5.** 58° [12.1A; Example 2] **6.** $3\frac{1}{7}$ m^2 [12.3A; Example 1] **7.** 150° [12.1C; Example 8] **8.** 15.85 ft [12.2B; Example 4] **9.** 13.748 [12.5A; Example 1] **10.** 9.747 ft [12.5B; Example 3] **11.** $10\frac{1}{8}$ ft^2 [12.3B; You Try It 2] **12.** $\angle a = 45°$; $\angle b = 135°$ [12.1C; Example 8] **13.** $1\frac{1}{5}$ ft [12.6A; Example 2] **14.** 90° and 50° [12.1B; Example 4] **15.** The width of the canal is 25 ft. [12.6B; Example 5] **16.** The amount of extra pizza is 113.04 in^2. [12.3C; Example 3] **17.** It will cost $1113.69 to carpet the area. [12.3C; You Try It 3] **18.** The length of the rafter is 15 ft. [12.5C; Example 4] **19.** The area is 103.82 ft^2. [12.3C; Example 3] **20. a.** The area of the floor of a cell was 45 ft^2. [12.3C; Example 3] **b.** The volume of a cell was 315 ft^3. [12.4C; Example 7]

CUMULATIVE REVIEW EXERCISES

1. 48 [2.1B] **2.** $7\frac{41}{48}$ [2.4C] **3.** $\frac{39}{56}$ [2.7B] **4.** $\frac{2}{15}$ [2.8B] **5.** $-\frac{1}{24}$ [10.4A] **6.** $17.44/h [4.2B] **7.** 37.5 [4.3B] **8.** $\frac{3}{8}$ [5.1A] **9.** −4 [11.1A] **10.** 85 [5.4A] **11.** −6 [11.3A] **12.** $\frac{4}{3}$ [11.4B] **13.** 32,500 m [9.1A] **14.** 31.58 m [9.1A] **15.** −15 [11.2C] **16.** 2 [11.4B] **17.** The monthly payment is $708. [1.5D] **18.** The sales tax on the home theater system is $47.06. [4.3C] **19.** The original hourly wage was $29.20. [5.4B] **20.** The sale price of the window is $108. [6.2D] **21.** The value of the investment after 20 years will be $101,366.50. [6.3C] **22.** The weight of the package is 54 lb. [8.2C] **23.** The distance between the rivets is 60 cm. [9.1B] **24.** The number is −2. [11.6A] **25.** **a.** 74° **b.** 106° [12.1C] **26.** 29.42 cm [12.2B] **27.** 50 in^2 [12.3B] **28.** 92.86 in^3 [12.4B] **29.** 10.63 ft [12.5B] **30.** 36 cm [12.6B]

FINAL EXAM

1. 3259 [1.3B] **2.** 53 [1.5C] **3.** 60,205 [1.3B] **4.** 16 [1.6B] **5.** 144 [2.1A] **6.** $1\frac{49}{120}$ [2.4B] **7.** $3\frac{29}{48}$ [2.5C] **8.** $6\frac{3}{14}$ [2.6B] **9.** $\frac{4}{9}$ [2.7B] **10.** $\frac{1}{6}$ [2.8B] **11.** $\frac{1}{13}$ [2.8B] **12.** 164.177 [3.2A] **13.** 0.027918 [3.4A] **14.** 0.69 [3.5A] **15.** $\frac{9}{20}$ [3.6B] **16.** 24.5 mi/gal [4.2B] **17.** 54.9 [4.3B] **18.** $\frac{9}{40}$ [5.1A] **19.** 135% [5.1B] **20.** 125% [5.1B] **21.** 36 [5.2A] **22.** $133\frac{1}{3}\%$ [5.3A] **23.** 70 [5.4A] **24.** 20 in. [8.1A] **25.** 1 ft 4 in. [8.1B] **26.** 2.5 lb [8.2A] **27.** 6 lb 6 oz [8.2B] **28.** 2.25 gal [8.3A] **29.** 1 gal 3 qt [8.3B] **30.** 248 cm [9.1A] **31.** 4.62 m [9.1A] **32.** 1.614 kg [9.2A] **33.** 2067 ml [9.3A] **34.** 88.55 km [9.5A] **35.** The cost is $1.15. [9.4A] **36.** 6.79×10^{-8} [10.5A] **37.** 3.9 m [12.2A] **38.** 45 in^2 [12.3A] **39.** 1200 cm^3 [12.4A] **40.** −4 [10.2A] **41.** −15 [10.2B] **42.** $-\frac{1}{2}$ [10.4B] **43.** $-\frac{1}{4}$ [10.4B] **44.** 6 [10.5B] **45.** $-x + 17$ [11.1C] **46.** −18 [11.2C] **47.** 5 [11.3A] **48.** 1 [11.4A] **49.** Your new balance is $959.93. [6.7A] **50.** 63,750 people will vote. [4.3C] **51.** One year ago, the dividend per share was $2.00. [5.4B] **52.** The average monthly income is $3794. [7.4A] **53.** The simple interest due is $7200. [6.3A] **54.** The probability is $\frac{1}{3}$. [7.5A] **55.** The death count of China is 6.7% of the death count of the four countries. [7.1B] **56.** The discount rate for the Bose headphones is 28%. [6.2D] **57.** The weight of the box is 81 lb. [8.2C] **58.** The perimeter is approximately 28.56 in. [12.2B] **59.** The area is approximately 16.86 cm^2. [12.3B] **60.** The number is 16. [11.6A]